Coefficient of Friction

OBJECTIVES

- ❑ Determine μ_s, the static coefficient of friction, and μ_k, the kinetic coefficient of friction, by sliding the block down a board that acts as an inclined plane.
- ❑ Determine μ_s and μ_k using the board with a pulley mounted on it in a horizontal position and applying known forces to the block.
- ❑ Compare the different values for μ_s and μ_k obtained by the two different techniques.
- ❑ Demonstrate that the coefficients are independent of the normal force, and that $\mu_s > \mu_k$.

EQUIPMENT LIST

- Smooth wooden board with pulley attached to one end
- Smooth wooden block (6-inch long 2×4 for example) with hook attached
- Second wooden block of known length to form inclined plane
- String, balance, calibrated masses, mass holder, and slotted masses

THEORY

Friction is a resisting force that acts along the tangent to two surfaces in contact when one body slides or attempts to slide across another. **Normal force** is the force that each body exerts on the other body, and it acts perpendicular to each surface. The frictional force is directly proportional to the normal force.

There are two different kinds of friction. **Static friction** occurs when two surfaces are still at rest with respect to each other, but an attempt is being made to cause one of them to slide over the other one. Static friction arises to oppose any force trying to cause motion tangent to the surfaces. The static frictional force f_s is given by

$$f_s \leq \mu_s N \qquad \text{(Eq. 1)}$$

where N stands for the normal force between the two surfaces, and μ_s is a constant called the **coefficient of static friction**. The meaning of Equation 1 is that the static frictional force varies in response to applied forces from zero up to a maximum value given by the equality in that equation. If the applied force is less than the maximum, then the frictional force that arises is equal to the applied force, and there is no motion. If the applied force is greater than the maximum, the object will begin to move, and static friction conditions are no longer valid.

The other kind of friction occurs when two surfaces are moving with respect to each other. It is called **kinetic friction,** and it is characterized by a constant μ_k called the **coefficient of kinetic friction**. The kinetic frictional force f_k is given by

$$f_k = \mu_k N \qquad \text{(Eq. 2)}$$

where N is again the normal force. Equation 2 states that the kinetic frictional force is a constant value any time the object is in motion. In fact, the coefficient of kinetic friction does vary somewhat with speed. It is assumed for this laboratory that at the slow speeds used, μ_k does not depend upon speed. To a good approximation both coefficients are independent of the apparent area of contact between the two surfaces.

An inclined plane with an angle θ that can be adjusted is shown in Figure 7-1. If a block is placed on the plane, and the angle is slowly increased, the block will begin to slip at some angle. The normal force N acts perpendicularly to the plane, and a component of the weight of the block $mg \cos \theta_s$ acts in the opposite direction. The block is in equilibrium for motion perpendicular to the plane, and these forces are equal and

$$N = mg \cos \theta_s \qquad \text{(Eq. 3)}$$

where θ_s is the angle at which the block just begins to slip on the inclined plane. Parallel to the plane there are also two forces. A component of the weight of the block $mg \sin \theta_s$ acts down the plane, and the frictional force f_s acts up the plane. At the point where the block just slips, the maximum frictional force is exerted, and these two forces are equal. In equation form

$$f_s = \mu_s N = mg \sin \theta_s \qquad \text{(Eq. 4)}$$

Combining Equations 3 and 4 leads to

$$\mu_s = \frac{mg \sin \theta_s}{N} = \frac{mg \sin \theta_s}{mg \cos \theta_2} = \tan \theta_s \qquad \text{(Eq. 5)}$$

Equation 5 can be used to determine μ_s by measuring $\tan \theta_s$ when the block just begins to slip on the inclined plane.

In a similar way the same inclined plane can be used to determine μ_k. By giving the block a slight push to get it started, the angle can be determined at which the block slides down the plane at constant velocity. A push is needed to get it started because generally $\mu_s > \mu_k$, and the static frictional force is greater than the

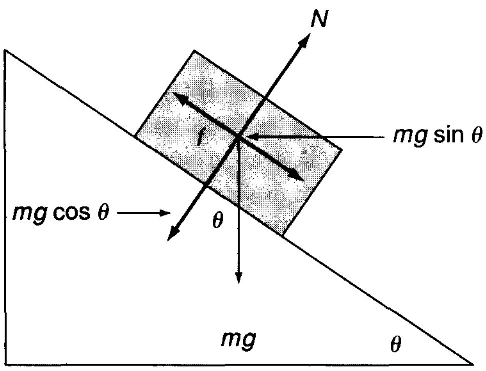

Figure 7-1 Forces acting on a block on an inclined plane.

kinetic frictional force. When the block is moving down the plane at constant velocity, the block is in equilibrium with the vector sum of forces on the block equal to zero. In equation form

$$N = mg\cos\theta_k \qquad \text{and} \qquad f_k = mg\sin\theta_k = \mu_k N \qquad \text{(Eq. 6)}$$

Using algebra to combine these equations leads to

$$\mu_k = \frac{mg\sin\theta_k}{N} = \frac{mg\sin\theta_k}{mg\cos\theta_k} = \tan\theta_k \qquad \text{(Eq. 7)}$$

Equation 7 can be used to determine μ_k from the angle at which the block slides down the inclined plane at constant velocity after it has been given a slight push to get it started.

If the inclined board is lowered to the horizontal position, a force can be applied to the block by means of a string running over a pulley and down to a mass as shown in Figure 7-2. For a given block mass M_1 it is possible to slowly add mass to M_2 until M_1 moves. The point at which the block just moves is the last point that the system is in equilibrium, and it occurs when the maximum static friction is acting. At that point the following conditions are met:

$$T = f_s \qquad T = M_2 g \qquad N = M_1 g \qquad f_s = \mu_s N \qquad \text{(Eq. 8)}$$

In these equations T is the tension in the string, and the other symbols are the same as previously defined. Combining these four equations leads to

$$f_s = M_2 g = \mu_s N = \mu_s M_1 g \qquad \text{(Eq. 9)}$$

Using the second and fourth terms above and canceling the common factor of g gives

$$M_2 = \mu_s M_1 \qquad \text{(Eq. 10)}$$

Equation 10 can be used to determine μ_s by finding the minimum mass M_2 needed to cause the block of mass M_1 to just move.

The same procedure can also be used to determine the coefficient of kinetic friction. Refer to Figure 7-2 and imagine that f_s is replaced by f_k for the case when the block is in motion. When the system is moving at constant velocity it is also in equilibrium, and the following conditions are met:

$$T = f_k \qquad T = M_2 g \qquad N = M_1 g \qquad f_k = \mu_k N \qquad \text{(Eq. 11)}$$

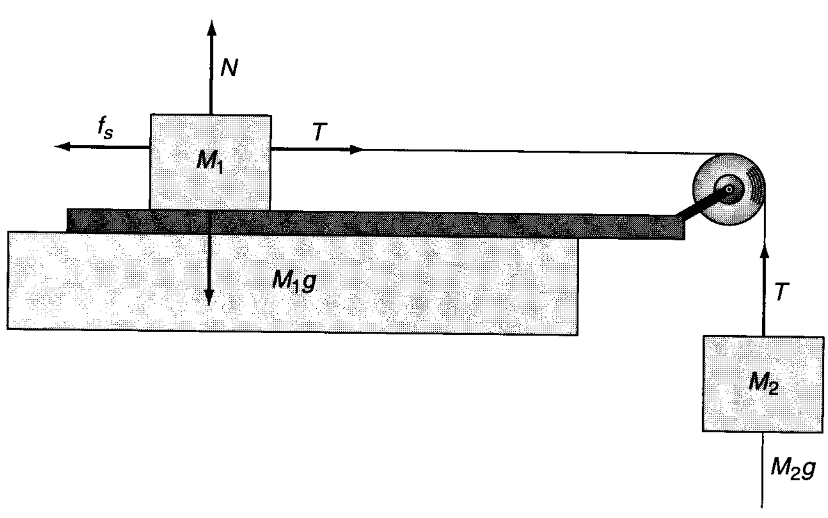

Figure 7-2 Force applied to a block on a horizontal plane.

Combining these equations leads to

$$M_2 = \mu_k M_1 \qquad \text{(Eq. 12)}$$

Equation 12 can be used to measure μ_k by finding the value of M_2 needed to cause M_1 to move at constant velocity.

EXPERIMENTAL PROCEDURE

The Inclined Plane

1. *Note which side of the block and which side of the board you are using. Continue to use the same surfaces for all the measurements made in this laboratory.*
2. Place the block with the hook attached with its large surface down on the board and incline the board until the block just begins to slide on its own. Incline the board by placing the block of known height (approximately 15.0 cm) under one end of the board. Record the value of the height of the block as Y (to the nearest 1 mm, which is 0.001 m) in both Data Table 1 and Data Table 2. Move the block of known height Y toward the line along which the board is resting on the table. This will increase the angle θ as shown in Figure 7-3.
3. When the block on top slides down the board because static friction can no longer hold it in place, record in Data Table 1 (to the nearest 0.001 m) the value of X, the distance from the pivot line of the board to the block as shown in Figure 7-3.
4. Repeat Step 3 three more times for a total of four trials with only the block itself on top of the board. Record the values of X associated with each of these trials in Data Table 1 in the column labeled 0 mass added.
5. Using light tape, attach a 0.200 kg mass to the top of the block and repeat the steps above, recording in Data Table 1 the values of X for the four trials in the column labeled 0.200 kg added.
6. Continue the process adding 0.400 kg and then 0.600 kg, and record the values of X for four trials in each case in the appropriate column in Data Table 1. The data taken in these first six steps will determine μ_s, from the fact that $\mu_s = \tan\theta_s = Y/X$.
7. Repeat all of the procedures above again, but instead of finding the point at which the block begins to move on its own, find the point at which the block moves at approximately constant speed after it is given a slight push to begin its motion. There will probably be more variation in the values of X obtained in this case because it is difficult to determine when the speed is constant. Again start with the block alone, and then add the same values of mass as above. In each case do four trials and record the values of X in the appropriate columns in Data Table 2. The data taken in these steps will be used to determine the coefficient of kinetic friction, μ_k.

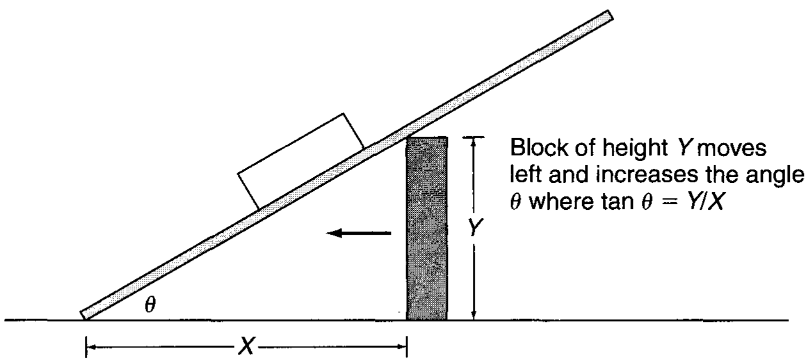

Figure 7-3 Geometry to increase θ by moving block of height Y.

Horizontal Plane with Pulley

1. Determine the mass of the block with hook attached using the balance. Record it in Data Table 3 as M_1 in the space labeled 0 kg added.
2. Place the board in a horizontal position on the laboratory table with the pulley beyond the edge of the table as shown in Figure 7-2.
3. Attach a piece of string to the hook in the block. Place it over the pulley and attach the mass holder to the other end of the string. *Be sure that the same surface of the block and board are in contact as in the first procedure.* Add mass to the mass holder to find the minimum mass needed to just cause the block to move. Record the value as M_2 in Data Table 3. Include the 0.050 kilogram mass of the holder in the total for M_2. Repeat the procedure two more times for a total of three trials.
4. Repeat Step 3 but add 0.200 kg to the top of the block. Record the value of the mass of the block plus 0.200 kg as M_1. Again determine the minimum mass needed to just cause the mass M_1 to move. Do three trials and record each as M_2 in Data Table 3.
5. Continue this process adding 0.400, 0.600, and finally 0.800 kg to the top of the block. In each case take three trials.
6. Perform a similar set of measurements as just described in Steps 1 through 5, but this time determine the mass M_2 needed to keep the block moving at constant velocity after it has been started with a small push. Again take three trials for each case and use values of M_1 beginning with the mass of the block and increasing in steps of 0.200 kg up to a total added mass of 0.800 kg. Record the values of M_2 and M_1 for all cases in Data Table 4.

CALCULATIONS

Board as an Inclined Plane

1. For the static friction data calculate the value of the mean $\overline{X}$ for the four trials of X at each value of M_1 and record them in Calculations Table 1 under the Static section. For each of the values of $\overline{X}$ in Calculations Table 1 calculate the value of μ_s as $\tan\theta_s = Y/\overline{X}$ and record them in Calculations Table 1 under the Static section. Calculate $\overline{\mu}_s$ and the standard error $\alpha_{\mu s}$ for the four measurements. Record those results.
2. For the kinetic friction data calculate the value of the mean $\overline{X}$ for the four trials of X at each value of M_1 and record them in Calculations Table 1 under the Kinetic section. For each of the values of $\overline{X}$ calculate the value of μ_k as $\tan\theta_k = Y/X$ and record them in Calculations Table 1 under the Kinetic section. Calculate $\overline{\mu_k}$ and the standard error $\alpha_{\mu k}$ for the four measurements. Record those results.

Horizontal Plane with Pulley

1. Calculate the mean $\overline{M_2}$ for the three trials of M_2 for each of the values of M_1 for both the static and kinetic friction cases. Record these values in Calculations Table 2 along with the value of M_1 for each value of $\overline{M_2}$.
2. According to Equation 10 there is a linear relationship between M_2 and M_1. A linear least squares fit to the static friction values of $\overline{M_2}$ versus M_1 should produce a straight line with a slope of μ_s. Perform such a fit with $\overline{M_2}$ as the vertical axis and M_1 as the horizontal axis. Record the value of the slope as μ_s in Calculations Table 2 under the Static section. Record the value of r.
3. Equation 12 states that there should also be a linear relationship between M_2 and M_1 for the kinetic friction data. Perform a linear least squares fit with $\overline{M_2}$ as the vertical axis and M_1 as the horizontal axis. Record the value of the slope as μ_k in Calculations Table 2 under the Kinetic section. Record the value of r.

GRAPHS

1. Graph the static friction data for the horizontal plane case with $\overline{M_2}$ as the vertical axis and M_1 as the horizontal axis. Show the straight line obtained from the fit.
2. Graph the kinetic friction data for the horizontal plane case with $\overline{M_2}$ as the vertical axis and M_1 as the horizontal axis. Show the straight line obtained from the fit.

Name

Section

Date

LABORATORY 7 *Coefficient of Friction*

PRE-LABORATORY ASSIGNMENT

1. For kinetic friction the direction of the frictional force on a given object is always opposite the direction of that object's motion. (a) true (b) false

2. The two coefficients of friction discussed in this laboratory are static (μ_s) and kinetic (μ_k). Describe the conditions under which each kind is appropriate. Generally, which of the two is larger?

3. Suppose a block of mass 25.0 kg rests on a horizontal plane, and the coefficient of static friction between the surfaces is 0.220. (a) What is the maximum possible static frictional force that could act on the block? ____________N (b) What is the actual static frictional force that acts on the block if an external force of 25.0 N acts horizontally on the block? ____________N Assume $g = 9.80\,\text{m/s}^2$. Show your work and explain both answers.

4. To measure the coefficient of kinetic friction by sliding a block down an inclined plane the block must be in equilibrium. What experimental condition must you try to accomplish that will assure you that the block is in equilibrium?

5. A 5.00 kg block rests on a horizontal plane. A force of 10.0 N applied horizontally causes the block to move horizontally at constant velocity. What is the coefficient of kinetic friction between the block and the plane? Assume $g = 9.80\ \text{m/sec}^2$. Show your work.

6. Both types of coefficient of friction are dimensionless. Why is this true?

7. For either type of coefficient of friction, what is generally assumed about the dependence of the value of the coefficient on the area of contact between the two surfaces?

Name ______ *Section* ______ *Date* ______

Lab Partners ______

LABORATORY 7 *Coefficient of Friction*

LABORATORY REPORT

Data Table 1 Inclined Plane—Static Friction

Y = ______ m

Added Mass	0.000 kg	0.200 kg	0.400 kg	0.600 kg
M_1 (kg)				
X Trial 1 (m)				
X Trial 2 (m)				
X Trial 3 (m)				
X Trial 4 (m)				

Data Table 2 Inclined Plane—Kinetic Friction

Y = ______ m

Added Mass	0.000 kg	0.200 kg	0.400 kg	0.600 kg
M_1 (kg)				
X Trial 1 (m)				
X Trial 2 (m)				
X Trial 3 (m)				
X Trial 4 (m)				

Data Table 3 Horizontal Plane—Static Friction

Added Mass	0.000 kg	0.200 kg	0.400 kg	0.600 kg	0.800 kg
M_1 (kg)					
M_2 Trial 1					
M_2 Trial 2					
M_2 Trial 3					

Data Table 4 Horizontal Plane—Kinetic Friction

Added Mass	0.000 kg	0.200 kg	0.400 kg	0.600 kg	0.800 kg
M_1 (kg)					
M_2 Trial 1					
M_2 Trial 2					
M_2 Trial 3					

Calculations Table 1—Inclined Plane

Static			
$\overline{X}$ (m)	$\mu_s = Y/\overline{X}$	$\overline{\mu}_s$	$\alpha_{\mu s}$

Kinetic			
$\overline{X}$ (m)	$\mu_k = Y/\overline{X}$	$\overline{\mu}_k$	$\alpha_{\mu k}$

Calculations Table 2—Horizontal Plane

Static			
M_1 (kg)	$\overline{M_2}$ (kg)	μ_s	r

Kinetic			
M_1 (kg)	$\overline{M_2}$ (kg)	μ_k	r

SAMPLE CALCULATIONS

1. $\overline{X} = (X_1+X_2+X_3+X_4)/4 =$
2. $\mu_s = Y/\overline{X} =$
3. Percent Difference for $\mu =$

QUESTIONS

1. Discuss the agreement between the two different measured values of μ_s. Calculate the percentage difference between them. Percentage difference = ____________%. Assume that the error in the horizontal plane value is approximately equal to the standard error for the inclined plane data. Under the assumption that both values of μ_s have that same error, do the two values of μ_s overlap within that assumed error? State the range of each measurement and quantitatively state the extent to which they overlap.

2. Answer the same questions as asked in Question 1, but now consider the kinetic friction data. Percentage difference = ____________%.

3. To what extent do your data confirm the expectation that the coefficients of friction, both static and kinetic, are independent of the normal force? What is the evidence for the inclined plane data? What is the evidence for the horizontal plane data? Give as quantitative an answer as possible in both cases.

4. Do your data confirm the expectation that $\mu_s \geq \mu_k$? Comment for both values of each coefficient and state your evidence.

5. State clearly in your own words what was to be accomplished in this laboratory. To what extent did your performance of the laboratory accomplish those goals?