# Nuclear Counting Statistics

## OBJECTIVES

- ❑ Investigate the counts from a radioactive source for 50 measurements under conditions in which the count rate should be approximately constant.
- ❑ Determine the standard deviation from the mean and standard error of the counts.
- ❑ Investigate how well the observed distribution of counts compares to that predicted by the normal distribution.
- ❑ Investigate whether or not $\sqrt{\overline{C}}$ approximates the standard deviation from the mean.

## EQUIPMENT LIST

- Geiger counter (single unit containing Geiger tube, power supply, timer, and scaler)
- Long-lived radioactive source (such as $^{137}Cs$ or $^{60}Co$)

## THEORY

If all other sources of error are removed from a nuclear counting experiment, there remains an uncertainty due to the random nature of the nuclear decay process. It is assumed that there exists some **true mean** value of the count, which shall be designated as $m$. But we emphasize, do not assume that there is a true value for any individual count $C_i$. Although $m$ is assumed to exist, it can never be known exactly. Instead, one can approach knowledge of the true mean $m$ by a large number of observations. It can be shown that the best approximation to the true mean $m$ is the mean $\overline{C}$, which is given by

$$\overline{C} = (1/n)\sum_{i=1}^{n} C_i \qquad \text{(Eq. 1)}$$

where $C_i$ stands for the ith value of the count obtained in $n$ trials. The **standard deviation from the mean** $\sigma_{n-1}$ and the **standard error** $\alpha$ are defined in the usual manner as

$$\sigma_{n-1} = \sqrt{\sum_{i=1}^{n}(1/n-1)(\overline{C}-C_i)^2} \quad \text{and} \quad \alpha = \frac{\sigma_{n-1}}{\sqrt{n}} \qquad \text{(Eq. 2)}$$

These equations have been applied to essentially all of the measurements in this laboratory manual. In many of the cases where these ideas have been applied, they are somewhat questionable because the

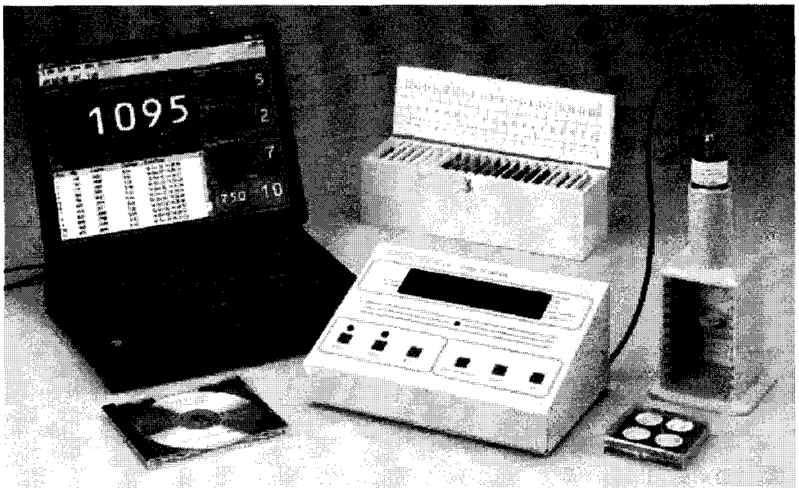


**Figure 46-1** Geiger counter with timer-scaler and encapsulated radioactive sources. (Photo courtesy of Sargent-Welch Scientific Co.)

random errors are not necessarily the determining factor. For nuclear counting experiments, usually the random errors are the limiting factor, and these concepts generally do apply strictly to such measurements.

The way in which the measurements $C_i$ are distributed around the mean $\overline{C}$ depends upon the statistical distribution. The binomial distribution is the fundamental law for the statistics of all random events including radioactive decay. Calculations are difficult with this distribution, and it is often approximated by another integral distribution called the Poisson distribution. For cases of $m$ greater than 20, both the binomial and the Poisson distribution can be approximated by the **normal distribution.** It has the advantage that it deals with continuous variables, and thus calculations are much easier with the normal distribution. For most nuclear counting problems of interest, the normal distribution predicts the same results for nuclear counting that have been assumed for measurements in general. Approximately 68.3% of the measured values of $C_i$ should fall within $\overline{C} \pm \sigma_{n-1}$, and approximately 95.5% of the measured values of $C_i$ should fall within $\overline{C} \pm 2\,\sigma_{n-1}$.

There is one statistical idea valid for nuclear counting experiments that is not true for measurements in general. For any given single measurement of the count $C$ in a nuclear counting experiment, an approximation to the standard deviation from the mean $\sigma_{n-1}$ is given by

$$\sigma_{n-1} \approx \sqrt{C} \qquad \text{(Eq. 3)}$$

For a series of repeated trials of a given count, the most accurate determination is given by $\overline{C} \pm \alpha$. If only a single measurement of the count is made, the most accurate statement that can be made is given by $C \pm \sqrt{C}$.

In this laboratory, we will take a series of measurements of the same count to determine the distribution of the measurements about the mean. In addition, we will investigate the validity of Equation 3.

## EXPERIMENTAL PROCEDURE

1. Consult your instructor for the operating voltage of the Geiger counter and set the Geiger counter to the proper operating voltage. Place a long-lived radioactive isotope on whichever counting shelf is necessary to produce between 500 and 700 counts in a 30 s counting period. For best results, the Geiger counter should have preset timing capabilities. If it does not and a laboratory timer is used, it would improve the timing precision if 60 s counting intervals are used. For whatever time is counted, between 500 and 700 counts should be recorded.

2. Repeat the count for a total of 50 trials. Make no changes whatsoever in the experimental arrangement for these 50 trials. Record each count in the Data Table. Do not make any background subtraction. Simply record the total count for each counting period.

## CALCULATIONS

1. Calculate the mean count $\overline{C}$, the standard deviation from the mean $\sigma_{n-1}$, and the standard error $\alpha$ for the 50 trials of the count and record the results in the Calculations Table.
2. For each count $C_i$ calculate $|C_i - \overline{C}|/\sigma_{n-1}$ and record the results in the Calculations Table.
3. Determine what percentage of the counts $C_i$ are further from $\overline{C}$ than $\sigma_{n-1}$ by counting the number of times a value of $|C_i - \overline{C}|/\sigma_{n-1} > 1$ occurs. Express this number divided by 50 as a percentage. Count the number of times that $|C_i - \overline{C}|/\sigma_{n-1} > 2$ occurs. Express this number divided by 50 as a percentage. Record these results in the Calculations Table.
4. Calculate $\sqrt{\overline{C}}$ and record its value in the Calculations Table.

## GRAPHS

1. Construct a histogram of your data on linear graph paper. Consider the range of the data and arbitrarily divide the range into about 15 segments. For counts in the range used, this should give intervals of 8 or 10 counts. An example of some data is displayed in this manner in Figure 46-2. The mean of the data is 659 with $\sigma_{n-1} = 27$, and an interval of 10 has been chosen.

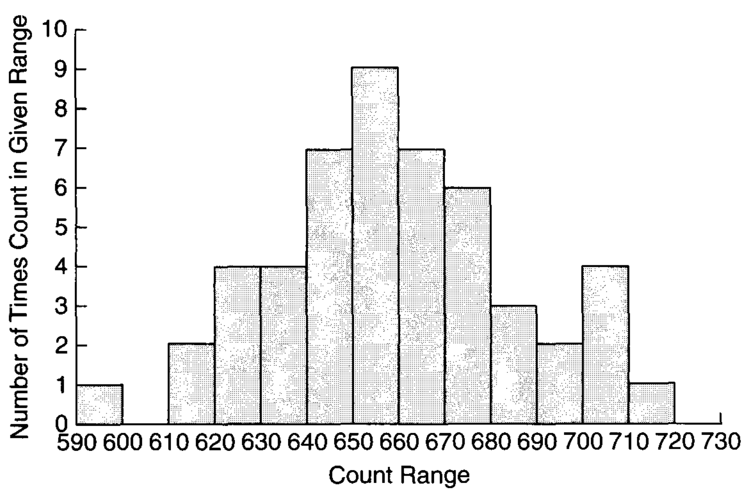


**Figure 46-2** Histogram of 50 repeated counts with mean of 659.

Name

Section

Date

# LABORATORY 46 *Nuclear Counting Statistics*

## PRE-LABORATORY ASSIGNMENT

1. For nuclear counting experiments no true value of a given count is assumed. What quantity is assumed to have a true value?

2. What is the exact statistical distribution function that describes the statistics of nuclear counting experiments?

3. What statistical distribution function approximates nuclear counting statistics and is used because it deals with continuous variables? For what values of the true mean $m$ is this distribution valid?

4. In a nuclear counting experiment a single measurement of $C$ counts is obtained. What is the approximate value for $\sigma_{n-1}$ for the count $C$?

5. According to the normal distribution function, when a given count is repeated 30 times, approximately how many of the results should fall in the range $\overline{C} \pm \sigma_{n-1}$? How many should fall in the range $\overline{C} \pm 2\sigma_{n-1}$?

6. A single count of a radioactive nucleus is made, and the result is 927 counts. What is the approximate value of $\sigma_{n-1}$?

7. A set of 10 repeated measurements of the count from a given radioactive sample was taken. The results were: 633, 666, 599, 651, 654, 690, 660, 659, 664, and 612. What is the mean count $\overline{C}$? What is the value of $\sigma_{n-1}$? What is the value of $\alpha$? Which of the counts fall outside $\overline{C} \pm \sigma_{n-1}$? Is this approximately the number of cases expected? Show your work.

8. For the data in Question 7, is $\sqrt{\overline{C}}$ approximately equal to $\sigma_{n-1}$? Calculate the percentage difference between the two. Show your work.

Name Section Date

Lab Partners

# 46 LABORATORY 46 Nuclear Counting Statistics

## LABORATORY REPORT

*Data Table*

| $i$ | $C_i$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |

| $i$ | $C_i$ |
|---|---|
| 18 | |
| 19 | |
| 20 | |
| 21 | |
| 22 | |
| 23 | |
| 24 | |
| 25 | |
| 26 | |
| 27 | |
| 28 | |
| 29 | |
| 30 | |
| 31 | |
| 32 | |
| 33 | |
| 34 | |

*Calculations Table*

| $i$ | $\lvert C_i - \overline{C} \rvert / \sigma_{n-1}$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |

| $i$ | $\lvert C_i - \overline{C} \rvert / \sigma_{n-1}$ |
|---|---|
| 18 | |
| 19 | |
| 20 | |
| 21 | |
| 22 | |
| 23 | |
| 24 | |
| 25 | |
| 26 | |
| 27 | |
| 28 | |
| 29 | |
| 30 | |
| 31 | |
| 32 | |
| 33 | |
| 34 | |

(Continued)

*(Continued)*

| $i$ | $C_i$ |
|---|---|
| 35 | |
| 36 | |
| 37 | |
| 38 | |
| 39 | |
| 40 | |
| 41 | |
| 42 | |

| $i$ | $C_i$ |
|---|---|
| 43 | |
| 44 | |
| 45 | |
| 46 | |
| 47 | |
| 48 | |
| 49 | |
| 50 | |

| $i$ | $\|C_i-\overline{C}\|/\sigma_{n-1}$ | $i$ | $\|C_i-\overline{C}\|/\sigma_{n-1}$ |
|---|---|---|---|
| 35 | | 43 | |
| 36 | | 44 | |
| 37 | | 45 | |
| 38 | | 46 | |
| 39 | | 47 | |
| 40 | | 48 | |
| 41 | | 49 | |
| 42 | | 50 | |
| $\overline{C}=$ | | $\sigma_{n-1}=$ | |
| $\alpha=$ | | $\sqrt{\overline{C}}=$ | |
| % trial $>\sigma_{n-1}$ from mean $=$ | | | |
| % trial $>2\ \sigma_{n-1}$ from mean $=$ | | | |

## SAMPLE CALCULATIONS

1. $|C_i-\overline{C}|/\sigma_{n-1}=$
2. $\sqrt{\overline{C}}=$
3. % trial $>\sigma_{n-1}$ from mean $=$
4. % trial $>2\ \sigma_{n-1}$ from mean $=$

## QUESTIONS

1. Consider the shape of the histogram of your data. Does it show the expected distribution relative to the mean of the data?

2. Compare the percentage of trials that have $|C_i - \overline{C}|/\sigma_{n-1} > 1$ with that predicted by the normal distribution. Compare the percentage of trials that have $|C_i - \overline{C}|/\sigma_{n-1} > 2$ with that predicted by the normal distribution.

3. What is the most accurate statement that you can make about the count from the sample based upon the data that you have taken?

4. Calculate the percentage difference between $\sqrt{\overline{C}}$ and $\sigma_{n-1}$. Do the results confirm the expectations of Equation 3?