Alternating Current RC *and* LCR *Circuits*

OBJECTIVES

- ❑ Investigate the phase relationship between the voltage across the resistor V_R and the voltage across the capacitor V_C in an RC circuit.
- ❑ Determine the value of the capacitance of a capacitor in an RC circuit.
- ❑ Investigate the phase relationships among the voltages across the resistor, the capacitor, and the inductor in an LCR circuit.

EQUIPMENT LIST

- Sine wave generator (variable frequency, 5 V peak to peak amplitude), resistance box
- A 100-mH inductor of known L and r, 1.00-μF capacitor, compass, protractor
- Alternating current voltmeter (digital readout, capable of measuring high frequency)
- *(This laboratory assumes that either Laboratory 36 has previously been performed, and that the values of the inductance (L) and resistance (r) for that inductor coil have been recorded and retained, or else an inductor with accurate known values of L and r is provided.)*

THEORY

RC Circuit

A series circuit consisting of a capacitor C, a resistor R, and a sine wave generator of frequency f is shown in Figure 37-1. Also shown in the figure is a **phasor diagram** for the generator voltage V, the voltage across the resistor V_R, and the capacitor voltage V_C. It is assumed that all voltages discussed in this laboratory are root-mean-square values. The voltages V_R and V_C are 90° out of phase, and the voltages V, V_R, and V_C form a right triangle as shown in Figure 37-1. Therefore, the equation relating the magnitudes of the measured voltages in an **RC circuit** is

$$V = \sqrt{V_C^2 + V_R^2} \qquad \text{(Eq. 1)}$$

Note that Equation 1 is valid for only a pure capacitor with no resistive component. If measurements on a real capacitor show agreement with Equation 1, it would indicate that the capacitor has no significant resistive component.

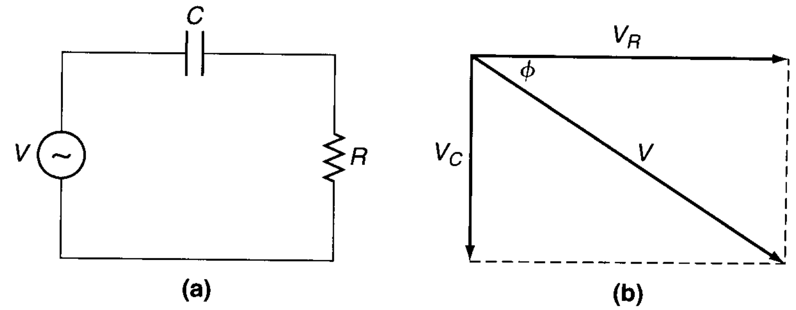

Figure 37-1 *RC* circuit and the phasor diagram for the voltage across each element.

In an *RC* circuit the current *I* is the same in each element of the circuit, and the relationships between the voltage and the current for the resistor and capacitor are

$$V_R = IR \quad \text{and} \quad V_C = I(1/\omega C) \qquad \text{(Eq. 2)}$$

The quantity $1/\omega C$ is called the **capacitive reactance,** and it has units of ohms. If the current is eliminated between the two equations in 2, an equation for *C* is given by

$$C = \left(\frac{1}{\omega R}\right)\left(\frac{V_R}{V_C}\right) \qquad \text{(Eq. 3)}$$

Thus a value for the capacitance of an unknown capacitor can be determined from Equation 3 if ω and *R* are known and V_R and V_C are measured.

LCR Circuit

Consider a series **LCR** circuit shown in Figure 37-2 with a generator of voltage *V*, a resistor *R*, a capacitor *C*, and an inductor having inductance *L* and resistance *r*. Note that the capacitor is assumed to have no resistance. Also shown in Figure 37-2 is the phasor diagram for the voltages V, V_R, V_C, V_L, and V_r. The figure shows that V_L and V_C are 180° out of phase, and V_R and V_r are in phase. The quantities $V_L - V_C$, V, and $V_R + V_r$ form a right triangle and

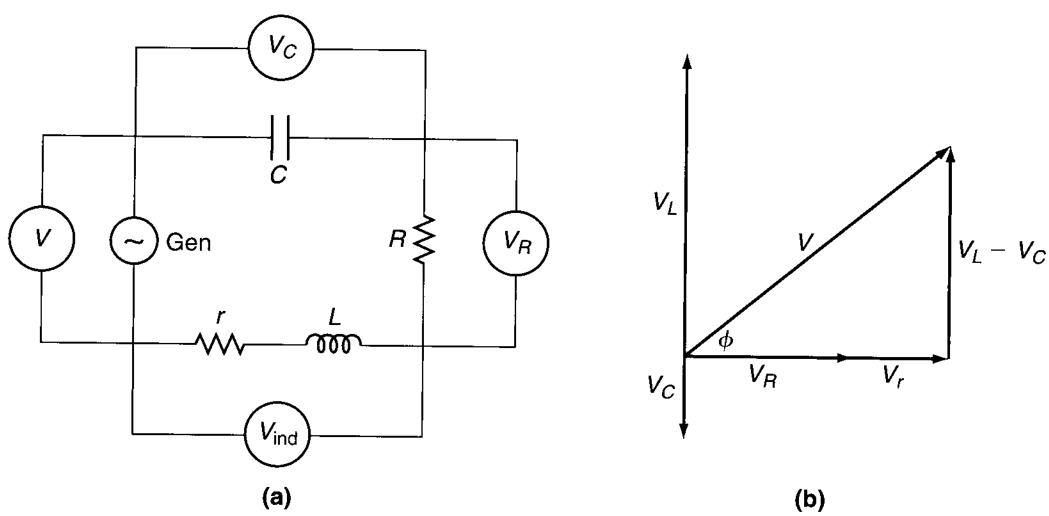

Figure 37-2 *LCR* circuit with voltmeter in the four positions to measure voltage across each element of the circuit. Also shown is a phasor diagram of all the relevant voltages.

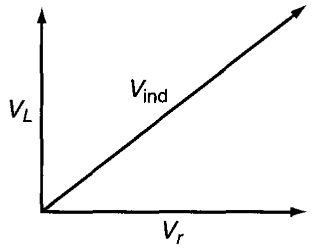

Figure 37-3 Phasor diagram for the voltages across the components of the inductor.

$$V = \sqrt{(V_L - V_C)^2 + (V_R + V_r)^2} \quad \text{(Eq. 4)}$$

It is not possible to measure either V_L or V_r directly. The only voltage associated with the inductor that can be measured experimentally is shown in Figure 37-2 as V_{ind}, and it is the vector sum of the voltages V_L and V_r. The relationship between V_{ind}, V_L, and V_r is shown in Figure 37-3. The figure shows that the voltages V_{ind}, V_L, and V_r obey the relationship

$$V_{ind}^2 = V_L^2 + V_r^2 \quad \text{(Eq. 5)}$$

The current I is the same in each element of the circuit, and V_L and V_r can be expressed as

$$V_L = I\,(\omega L) \qquad \text{and} \qquad V_r = I\,r \quad \text{(Eq. 6)}$$

The quantity ωL is called the inductive reactance. It has units of ohms. If Equations 5 and 6 are combined, and the current I is eliminated, it can be shown that

$$V_L = V_{ind}\frac{\omega L}{\sqrt{(\omega L)^2 + r^2}} \qquad \text{and} \qquad V_r = V_{ind}\frac{r}{\sqrt{(\omega L)^2 + r^2}} \quad \text{(Eq. 7)}$$

Assuming that ω, L, and r are known, Equation 7 can be used to determine V_L and V_r if V_{ind} is measured. These values of V_L and V_r combined with measured values of V_R and V_C can be used in Equation 4 to verify the relationship between these quantities and the measured generator voltage V.

EXPERIMENTAL PROCEDURE

RC Circuit

1. Connect the capacitor provided in series with the sine wave generator and a resistance box to form a circuit like that shown in Figure 37-1. Set the generator output to maximum voltage and set the frequency to 250 Hz. Record the value of f in Data Table 1. Set the resistance box to a value of 300 Ω and record that value as R in Data Table 1.
2. Use the alternating voltage scale on the voltmeter to measure and record in the Data Table the generator voltage V, the capacitor voltage V_C, and the voltage across the resistor V_R.
3. Repeat the above procedure for three other cases using $R = 500$, 700, and 900 Ω. The generator output might change slightly in response to changes in R. Therefore, measure all three voltages for each value of R.
4. Obtain a value for the capacitance of your capacitor from your instructor. Record this known value as C_k in Data Table 1.

LCR Circuit

1. Construct a series *LCR* circuit like the one shown in Figure 37-2 using the same capacitor used previously, an inductor for which the values of L and r are known, a resistance box, and the sine wave generator. Record the values of L and r in Data Table 2.
2. Set the generator output to maximum voltage and set the frequency to 800 Hz. Set the resistance box to a value of 200 Ω. Record the values of f and R in Data Table 2.
3. Using the alternating voltage scale on the voltmeter, measure the generator voltage V, the capacitor voltage V_C, the resistor voltage V_R, and the inductor voltage V_{ind}. Record these values in Data Table 2.
4. Repeat the procedure of Steps 1 through 3 with the generator frequency set to $f = 600$ Hz and the resistance box set to $R = 200\,\Omega$.
5. Repeat the procedure of Steps 1 through 3 two more times, once with $R = 300\,\Omega$ and $f = 600$ Hz, and again with $R = 300\,\Omega$ and $f = 800$ Hz.

CALCULATIONS

RC Circuit

1. From the known value of the frequency f, calculate the value of the angular frequency $\omega = 2\pi f$. Record the value of ω in Calculations Table 1.
2. Calculate the quantity $\sqrt{V_C^2 + V_R^2}$ for each case and record the values in Calculations Table 1. Calculate the percentage error in each of these values compared to the measured values of the generator voltage and record them in Calculations Table 1.
3. Calculate the values of C from the measured values of V_R and V_C for each value of R. Record each value of C in Calculations Table 1. Calculate the mean $\overline{C}$ and the standard error α_C for the four values of C and record them in Calculations Table 1.
4. Calculate the percentage error in the value of $\overline{C}$ compared to the known value of the capacitance C_k.

LCR Circuit

1. From the known value of f for each case, calculate the angular frequency $\omega = 2\pi f$ and record the values in Calculations Table 2.
2. Calculate the four values of V_L and V_r and record them in Calculations Table 2.
3. Calculate and record in Calculations Table 2 the four values of $V_L - V_C$ and $V_R + V_r$.
4. For each of the four cases calculate and record in Calculations Table 2 the value of the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$.
5. Calculate the percentage error in the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$ compared to the measured value of the generator voltage V. Record the values of those percentage errors in Calculations Table 2.

Name Section Date

LABORATORY 37 *Alternating Current* RC *and* LCR *Circuits*

PRE-LABORATORY ASSIGNMENT

1. In a series *RC* circuit such as the one in Figure 37-1, the following phase relationship exists between the generator voltage V, the capacitor voltage V_C, and the resistor voltage V_R. (a) V and V_R are in phase, V_C lags V_R by 90° (b) V_C and V_R are at angle ϕ, V leads V_C by 90° (c) V_C lags V_R by 90°, V lags V_R by angle ϕ (d) V_R lags V_C by ϕ, V_C leads V by 90°.

2. If a series *RC* circuit has $V_R = 12.6$ V and $V_C = 10.7$ V, the generator voltage must be (a) 12.6 V (b) 23.3 V (c) 1.9 V (d) 16.5 V. Show your work.

3. A series *RC* circuit has $\omega = 2000$ rad/s and $R = 300\ \Omega$. The voltage V_C is measured to be 4.76 V, and V_R is measured to be 6.78 V. What is the value of C? Show your work. (a) 2.37 μF (b) 5.00 μF (c) 1.17 μF (d) 4.67 μF.

4. A series *RC* circuit of $R = 500\ \Omega$ and $C = 3.00\ \mu$F is measured to have $V_R = 8.07$ V and $V_C = 6.68$ V. What is the current I, and what is the value of ω the angular frequency? Show your work.

   $I =$ ____________ A $\qquad \omega =$ ____________ rad/s

5. A series *LCR* circuit consists of an inductor of inductance L and resistance r, a capacitor C, a resistor R, and a generator of voltage V. Mark as true or false the following statements concerning the relative phase of V, V_L, V_r, V_C, and V_R.

_____ 1. V_r and V_R are in phase.

_____ 2. V_L leads V_C by 90°.

_____ 3. V is at angle ϕ relative to V_R.

_____ 4. $V_L - V_C$ is in phase with V_r.

_____ 5. V_C lags V_R by 90°.

6. Measurements on the circuit described in Question 5 give $V_L = 10.76$ V, $V_C = 5.68$ V, $V_R = 6.32$ V, and $V_r = 3.75$ V. What is the generator voltage V? Show your work.

V = ______________ V

7. An inductor with $L = 150$ mH and $r = 200\ \Omega$ is in series with a capacitor, a resistor, and a generator of $\omega = 1000$ rad/s. The voltage across the inductor V_{ind} is measured to be 10.87 V, V_R is measured to be 4.65 V, and V_C is measured to be 5.96 V. What is the generator voltage V? (Hint—This is the measurement to be performed in this laboratory for *LCR* circuits. Use the appropriate equations to find V_L and V_r, and then use them and the values of V_R and V_C to calculate V.)

Name ____________________ Section __________ Date __________

Lab Partners ____________________

LABORATORY 37 *Alternating Current* RC *and* LCR *Circuits*

LABORATORY REPORT

Data Table 1

$f =$ Hz		$C_k =$ μF		
R (Ω)				
V_C (V)				
V_R (V)				
V (V)				

Calculations Table 1

$\omega =$ rad/s				
$\sqrt{V_C^2 + V_R^2}$				
% Error				
C (μF)				
$\overline{C} =$ μF	$\alpha_C =$ μF	% Error $\overline{C} =$		

Data Table 2

$r =$ Ω		$L =$ H		C = μF
f (Hz)				
R (Ω)				
V (V)				
V_{ind} (V)				
V_C (V)				
V_R (V)				

Calculations Table 2

ω (rad/s)				
V_L (V)				
V_r (V)				
$(V_L - V_C)$ (V)				
$(V_R + V_r)$ (V)				
$\sqrt{(V_L - V_C)^2 + (V_R - V_r)^2}$ (V)				
% Error compared to V				

SAMPLE CALCULATIONS

1. $\omega = 2\pi f =$
2. $V = \sqrt{V_C^2 + V_R^2} =$
3. $C = (1/\omega R)(V_R/V_C) =$
4. $V_L = (V_{\text{ind}})\left(\omega L/\sqrt{(\omega L)^2 + r^2}\right) =$
5. $V_r = (V_{\text{ind}})\left(r/\sqrt{(\omega L)^2 + r^2}\right) =$
6. $V = \sqrt{(V_L - V_C)^2 + (V_R + V_r)^2} =$

QUESTIONS

1. Comment on the agreement between the measured generator voltage V and the quantity $\sqrt{V_C^2 + V_R^2}$ for the RC circuit data.

2. Do your results for Question 1 confirm that the capacitor has no resistance? State specifically how the data either do or do not confirm this expectation.

3. State carefully your evaluation of the precision of your measurements of the value of the capacitor in the RC circuit. State the evidence for your opinion.

4. Considering the given value of C_k as the true value, comment on the accuracy of your measurements of the capacitance.

5. Comment on the agreement between the measured generator voltage V and the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$ in the LCR circuit.

6. Do your results confirm the phasor diagram of Figure 37-2 as a correct model for the addition of the voltages in an *LCR* circuit? State why they do or do not confirm this model.