Measurement of Electrical Resistance and Ohm's Law

OBJECTIVES

- ❑ Define the concept of electrical resistance using measurements of the voltage across and current in a wire coil.
- ❑ Investigate the dependence of the resistance on the length, cross-sectional area, and resistivity of the wire.
- ❑ Investigate the equivalent resistance of series and parallel resistors.

EQUIPMENT LIST

- Resistance coils (standard set available from Sargent-Welch or Central Scientific consisting of 10 m and 20 m length of copper and German silver wire)
- Direct current ammeter (0–2 A), direct current voltmeter (0–30 V, preferably digital readout)
- Direct current power supply (0–20 V at 1 A)

THEORY

If a **voltage** V is applied across an element in an electrical circuit, the **current** I in the element is determined by a quantity known as the **resistance** R. The relationship between these three quantities serves as a definition of resistance.

$$R = \frac{V}{I} \qquad \text{(Eq. 1)}$$

The units of resistance are volt/ampere, which are given the name ohm. The symbol for ohm is Ω. Some circuit elements obey a relationship known as **Ohm's Law.** For these elements the quantity R is a constant for different values of V. If a circuit element obeys Ohm's Law, when the voltage V is varied the current I will also vary, but the ratio V/I should remain constant. In this laboratory we will perform measurements on five coils of wire to investigate if they obey Ohm's Law. We also will determine the resistance of the coils.

The resistance of any object to electrical current is a function of the material from which it is constructed, the length, the cross-sectional area, and the temperature of the object. At constant temperature the resistance R is given by

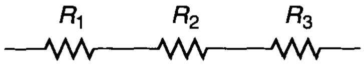

Figure 28-1 Resistors in series.

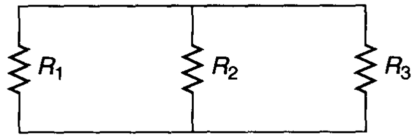

Figure 28-2 Resistors in parallel.

$$R = \rho \frac{L}{A} \qquad \text{(Eq. 2)}$$

where R is the resistance (Ω), L is the length (m), A is the cross-sectional area (m^2), and ρ is a constant dependent upon the material called the **resistivity** (Ω–m). Actually ρ is a function of temperature, and if the temperature of the coils of wire rises as a result of the current in them, this may be a source of error.

Circuit elements in an electrical circuit can be connected in series or parallel. Three resistors (R_1, R_2, and R_3) are connected in series as shown in Figure 28-1. For resistors in series the current is the same for all the resistors, but the voltage drop across each resistor is different. For resistors in series the equivalent resistance R_e of the three resistors is given by

$$R_e = R_1 + R_2 + R_3 \qquad \text{(Eq. 3)}$$

The same three resistors are shown connected in parallel in Figure 28-2. For resistors in parallel the current is different in each resistor, but the voltage across each resistor is the same. In this case the equivalent resistance R_e of the three resistors in terms of the individual resistors is given by

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{(Eq. 4)}$$

One of the objectives of this laboratory will be to observe the behavior of resistors in series and parallel.

EXPERIMENTAL PROCEDURE

1. Connect the ammeter A, the voltmeter V, and the power supply PS to the first resistor as shown in Figure 28-3. The basic circuit is the power supply in series with a resistor. To measure the current in the resistor, the ammeter is placed in series. To measure the voltage across the resistor, the voltmeter is placed in parallel.
2. Vary the current through resistor R_1 in steps of 0.250 A up to 1.000 A. For each specified value of the current, measure the voltage across the resistor and record the values in Data Table 1. The resistors will heat up and may be damaged by allowing current in them for long periods of time. Measurements should be made quickly at each value of the current. *APPLY VOLTAGE ONLY WHEN DATA ARE BEING TAKEN.*
3. Repeat Step 2 for each of the five resistors. For each resistor the ammeter must be in series with that resistor and the power supply, and the voltmeter must be in parallel with the resistor. Record all values in Data Table 1.
4. Connect the first four resistors in series to measure the equivalent resistance of the combination. Use two values of current, 0.500 A and 1.000 A, and measure the value of the voltage for each of these values of current. Record the voltage in Data Table 2.

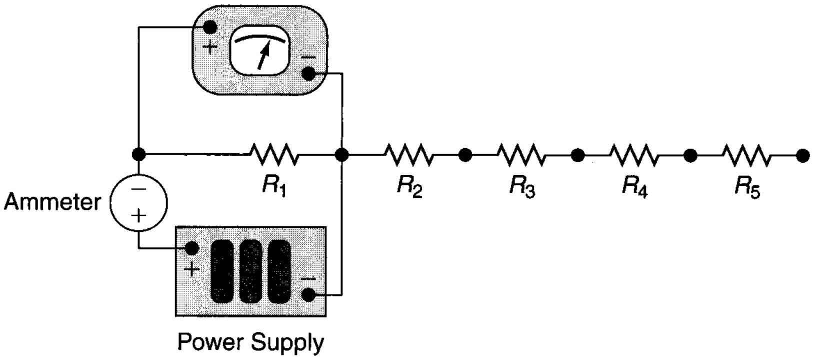

Figure 28-3 Measurement of current and voltage for resistor R_1.

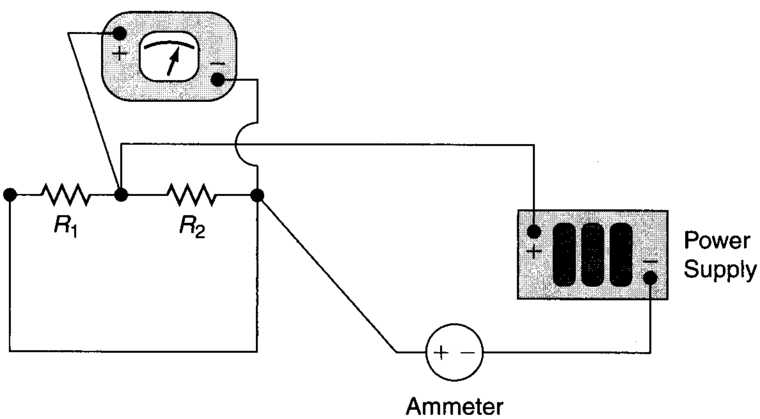

Figure 28-4 Resistors R_1 and R_2 in parallel.

5. Measure the voltage across the combination of R_2, R_3, and R_4 in series for currents of 0.500 A and 1.000 A and record the values in Data Table 2.
6. Connect R_1 and R_2 in parallel as shown in Figure 28-4 and measure the voltage across the combination for current values of 0.500 A and 1.000 A and record in Data Table 2.
7. Connect R_1 and R_3 in parallel as shown in Figure 28-5 and measure the voltage for current values of 0.500 A and 1.000 A and record in Data Table 2.
8. Connect R_2 and R_3 in parallel and perform the same measurements as described in Steps 6 and 7. Record the results in Data Table 2.

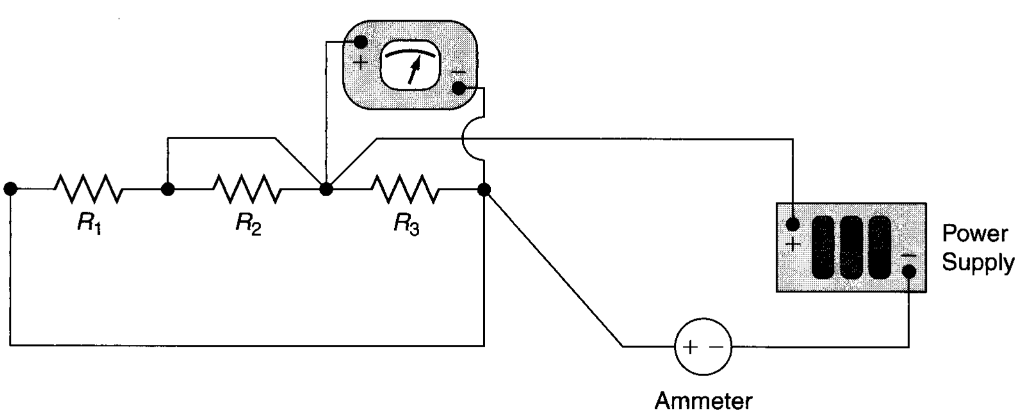

Figure 28-5 Resistors R_1 and R_3 in parallel.

CALCULATIONS

1. The first four coils are made of copper with resistivity of $\rho = 1.72 \times 10^{-8}$ Ω–m. The fifth coil is made of an alloy called German silver with resistivity of $\rho = 28.0 \times 10^{-8}$ Ω–m. The first, second, and fifth coils are 10.0 m long, and the third and fourth coils are 20.0 m long. The diameters of the first, third, and fifth coils are 0.0006439 m, and the diameters of the second and fourth coils are 0.0003211 m. Use these values in Equation 2 to calculate the value of the resistance for each of the five coils and record the results in Calculations Table 1 as the theoretical values for the resistance R_{theo}.
2. If Equation 1 is solved for V, the result is $V = IR$. There is a linear relationship between the voltage and the current, and the slope of V versus I will be the resistance R. Perform a linear least squares fit to the data in Data Table 1 with V as the vertical axis and I as the horizontal axis. Record in Calculations Table 1 the slope of the fit for each resistor as the experimental value for the resistance R_{exp}. Also record the value of the correlation coefficient r for each of the fits.
3. Calculate the percentage error in the values of R_{exp} compared to the values of R_{theo} for the five resistors and record the results in Calculations Table 1.
4. For the data of Data Table 2 calculate the values of the equivalent resistance for the various series and parallel combinations listed in the table as the value of the measured voltage divided by the appropriate current. Calculate and record the mean of the two trials as $(\overline{R_e})_{\text{exp}}$ in Calculations Table 2.
5. Equations 3 and 4 give the theoretical expressions for equivalent resistance for series and parallel combinations of resistance. Calculate a theoretical value for the equivalent resistance for each series and parallel combination measured in Data Table 2. For the values of the individual resistances R_1, R_2, and R_3 in Equation 3 and 4, use the experimental values determined from the fit to the data on the individual resistors. Record this theoretical value for the equivalent resistance in each case as $(R_e)_{\text{theo}}$ in Calculations Table 2.
6. Calculate the percentage difference between the values of $(\overline{R_e})_{\text{exp}}$ and $(R_e)_{\text{theo}}$ for each of the series and parallel combinations measured and record the results in Calculations Table 2.

GRAPHS

1. Construct graphs of the data in Data Table 1 with V as the vertical axis and I as the horizontal axis. Use only one piece of graph paper for all five resistors, making five small graphs on that one sheet. Choose different scales for each graph if needed, but make the five graphs as large as possible while still fitting on one page. Also show on each small graph the straight line for the linear least squares fit.

Name ______ Section ______ Date ______

LABORATORY 28 *Measurement of Electrical Resistance and Ohm's Law*

PRE-LABORATORY ASSIGNMENT

1. Three resistors R_1, R_2, and R_3 are connected in series with $R_1 < R_2 < R_3$. Choose all correct answers below. The total resistance of the combination is (a) less than R_1, (b) less than R_3, (c) greater than R_3, (d) greater than 3 R_3, (e) equal to $R_1 + R_2 + R_3$.

2. Two resistors R_1 and R_2 are connected in parallel with $R_1 < R_2$. Choose all correct answers below. The total resistance of the combination is (a) less than R_1, (b) less than R_2, (c) greater than R_2, (d) greater than 2 R_2, (e) equal to $(R_1R_2)/(R_1 + R_2)$.

3. A wire of length L_1 and diameter d_1 has resistance R_1. A second wire of the same material has length $L_2 = 2\ L_1$ and diameter $d_2 = 2\ d_1$. The resistance of wire two is R_2. Choose the correct value for R_2. (a) $R_2 = R_1$, (b) $R_2 = 2R_1$, (c) $R_2 = \frac{1}{2}R_1$, (d) $R_2 = 4R_1$.

4. If a circuit element carries a current of 3.71 A, and the voltage drop across the element is 8.69 V, what is the resistance of the circuit element? Show your work.

   $R =$ ____________________ Ω

5. A resistor is known to obey Ohm's Law. When there is a current of 1.72 A in the resistor, it has a voltage drop across its terminals of 7.35 V. If a voltage of 12.0 V is applied across the resistor, what is the current in the resistor? Show your work.

   $I =$ ____________________ A

6. The resistivity of copper is 1.72×10^{-8} Ω–m. A copper wire is 15.0 m long, and the wire diameter is 0.0500 cm. What is the resistance of the wire? Show your work.

   $R =$ ____________________ Ω

7. A wire of cross-sectional area 5.00×10^{-6} m^2 has a resistance of 1.75 Ω. What is the resistance of a wire of the same material and length as the first wire, but with a cross-sectional area of 8.75×10^{-6} m^2? Show your work.

   $R =$ ____________________ Ω

8. Three resistors of resistance 20.0 Ω, 30.0 Ω, and 40.0 Ω are connected in series. What is their equivalent resistance? Show your work.

   $R =$ ____________________ Ω

9. Three resistors of resistance 15.0 Ω, 25.0 Ω, and 35.0 Ω are connected in parallel. What is their equivalent resistance? Show your work.

   $R =$ ____________________ Ω

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LABORATORY 28 *Measurement of Electrical Resistance and Ohm's Law*

LABORATORY REPORT

Data Table 1

I (A)	V_{R1} (V)	V_{R2} (V)	V_{R3} (V)	V_{R4} (V)	V_{R5} (V)
0.250					
0.500					
0.750					
1.000					

Calculations Table 1

	R_1	R_2	R_3	R_4	R_5
R_{theo}					
R_{exp}					
r					
% Error R_{exp}					

Data Table 2

Combination	I (A)	V (V)	I (A)	V (V)
R_1 R_2 R_3 R_4 Series	0.500		1.000	
R_2 R_3 R_4 Series	0.500		1.000	
R_1 R_2 Parallel	0.500		1.000	
R_1 R_3 Parallel	0.500		1.000	
R_2 R_3 Parallel	0.500		1.000	

Calculations Table 2

Combination	$(R_e)_{exp}{}^1$ (Ω)	$(R_e)_{exp}{}^2$ (Ω)	$(\overline{R_e})_{exp}$ (Ω)	$(R_e)_{theo}$ (Ω)	% *Diff*
R_1 R_2 R_3 R_4 Series					
R_2 R_3 R_4 Series					
R_1 R_2 Parallel					
R_1 R_3 Parallel					
R_2 R_3 Parallel					

SAMPLE CALCULATIONS

1. $R_{theo} = \rho L/A =$
2. $(R_e)_{exp} = V/I =$
3. % Error =
4. (Series) $(R_e)_{theo} = R_1 + R_2 + R_3 + R_4 =$
5. (Parallel) $(R_e)_{theo} = (1)/(1/R_1 + 1/R_2) =$
6. % Difference =

QUESTIONS

1. Do the individual resistors you have measured obey Ohm's Law? In answering this question, consider the least squares fits and the graphs you have made for each resistor. Remember that linear behavior of V versus I is the proof of ohmic behavior.

2. Evaluate the agreement between the theoretical values for the individual resistances and the experimental values.

3. Does your agreement between the experimental and theoretical values of the series combinations of resistors support Equation 3 as the model for series combination of resistors? The agreement is not expected to be perfect, but you are to determine if the agreement is reasonable within the expected experimental uncertainty.

4. Evaluate the agreement between the experimental and theoretical values of the parallel combinations of resistors. Do the results support Equation 4 as the model for the parallel combination of resistors within the expected experimental uncertainty?

5. The first and second coils have the same length, and the third and fourth coils have the same length. They differ only in the cross-sectional area. According to theory, what should be the ratio of the resistance of the second coil to the first and the fourth coil to the third? Calculate these ratios for your experimental results and compare the agreement with the expected ratio.

6. The first and third coils have the same cross-sectional area, and the second and fourth coils have the same cross-sectional area. They differ only in length. According to theory, what should be the ratio of the resistance of the third coil to the first and the fourth coil to the second? Calculate these ratios for your experimental results and compare the agreement with the expected ratio.