Archimedes' Principle

OBJECTIVES

- ❑ Apply Archimedes' principle to measurements of specific gravity.
- ❑ Determine the specific gravity of several metal objects that are denser than water, a cork that is less dense than water, and a liquid (alcohol).

EQUIPMENT LIST

- Laboratory balance, calibrated masses, 1000 mL beaker, string, alcohol
- Metal cylinders, and cork or piece of wood (to serve as unknowns)

THEORY

If the mass of an object is distributed uniformly, its **density** ρ_o is defined as the mass m of the object divided by its volume V. In equation form this is

$$\rho_o = m/V \qquad \text{(Eq. 1)}$$

The SI units for density are kg/m^3. Other commonly used units for density are the cgs system units g/cm^3. The **specific gravity** is defined as the ratio of the density of an object to the density of water ρ_w. The equation for the specific gravity is given by

$$\text{Specific gravity} = SG = \rho_o/\rho_w \qquad \text{(Eq. 2)}$$

Specific gravity is a dimensionless quantity because it is the ratio of two densities. The density of water in the cgs system is $1.000\ g/cm^3$, and densities in that system are numerically equal to the specific gravity SG. Water has a density of $1000\ kg/m^3$, and densities in the SI system are equal to $SG \times 10^3$.

Archimedes' principle states that an object placed in a fluid experiences an upward buoyant force equal to the weight of fluid displaced by the object. The principle applies to both liquids and gases. In this laboratory, we will examine the application of the principle to liquids. An object floats if its density is less than the density of the liquid in which it is placed. It sinks in the liquid until it displaces a weight of liquid equal to its own weight. The object is in equilibrium because its weight acts downward and a buoyant force acts upward. An object with density greater than the liquid in which it is placed will sink to the bottom of the liquid. It experiences an upward buoyant force equal to the weight of the displaced fluid, but that is less than the weight of the object, and it sinks.

For objects with a density greater than the density of water, Archimedes' principle allows a simple determination of the specific gravity of the object. Consider an object that is tied to a string and submerged in water with the string attached beneath the arm of a laboratory balance as shown in Figure 18-1. Also in the figure is a free-body diagram of the forces acting on the submerged object. If B stands for the buoyant force on the object, W is the weight of the object, and W_1 is the tension in the string, the following is true at equilibrium

$$B + W_1 = W \qquad \text{(Eq. 3)}$$

The quantity W_1 is the apparent weight read by the laboratory balance in Figure 18-1. By Archimedes' principle, the buoyant force is the weight of displaced water. This can be written as

$$B = m_w g = \rho_w V_w g = \rho_w V_o g \qquad \text{(Eq. 4)}$$

with m_w the mass of the displaced water, and V_w and V_o the volumes of the displaced water and the object. The last step in Equation 4 is true because the object sinks and displaces water equal to its volume, and therefore $V_w = V_o$. Using Equations 3 and 4 with $W = mg$ and $W_1 = m_1 g$ gives

$$W_1 = W - B = mg - \rho_w V_w g = \rho_o V_o g - \rho_w V_o g \qquad \text{(Eq. 5)}$$

$$\frac{W}{W - W_1} = \frac{mg}{mg - m_1 g} = \frac{\rho_o V_o g}{\rho_o V_o g - (\rho_o V_o g - \rho_w V_o g)} = \frac{\rho_o}{\rho_w} \qquad \text{(Eq. 6)}$$

$$SG = \frac{\rho_o}{\rho_w} = \frac{m}{m - m_1} \qquad \text{(Eq. 7)}$$

This laboratory will use Equation 7 to determine the specific gravity of several metals with densities greater than that of water.

To use Archimedes' principle to determine the specific gravity of an object that floats, the object must be submerged by attaching a lead weight to the object. Figure 18-2(a) shows the lead weight in water and the object in air, and the balance reading is W_1. In Figure 18-2(b) both the object and the lead weight are below the water, and the balance reads W_2. By analysis similar to that used to derive Equation 7, it can be shown that the specific gravity of an object that floats in water is given by

$$SG = \frac{W}{W_1 - W_2} = \frac{mg}{m_1 g - m_2 g} = \frac{m}{m_1 - m_2} \qquad \text{(Eq. 8)}$$

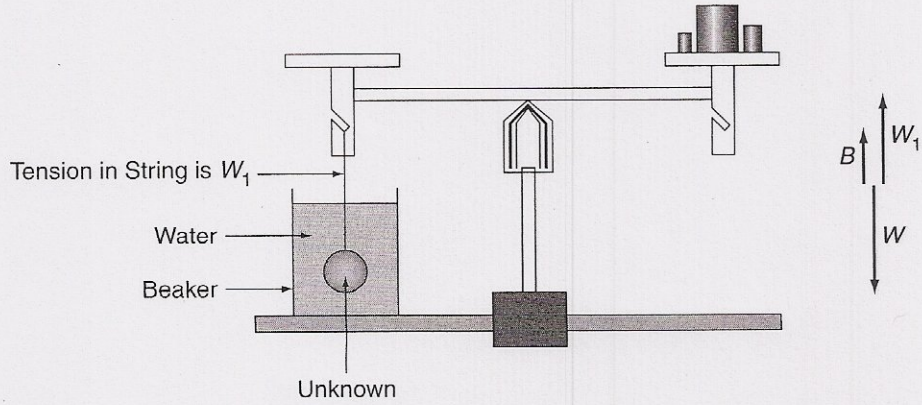

Figure 18-1 Determining the apparent mass of an object with density greater than water.

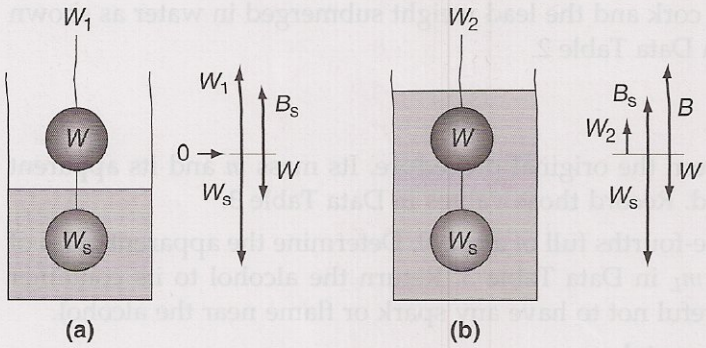

Figure 18-2 Forces acting on an object held submerged by a lead weight.

In Equation 8 m is the mass of the object, m_1 is the apparent mass with the object in air and the lead weight submerged in water, and m_2 is the apparent mass with the object and the lead weight both submerged in water.

The specific gravity of a liquid can be determined by measuring the mass of some object m, the apparent mass of the object submerged in water m_w, and the apparent mass of the object submerged in the liquid m_L. It is assumed that the object sinks in both liquids. By similar analysis as used to derive Equation 7, it can be shown that the specific gravity of the liquid is given by

$$SG(\text{liquid}) = \frac{m - m_L}{m - m_w} \qquad \text{(Eq. 9)}$$

EXPERIMENTAL PROCEDURE

Object Density Greater than Water

1. Use the laboratory balance to determine the mass of each of the two unknown metal cylinders. Record these values as m in Data Table 1. The unknowns should have their metal type stamped on them, or else ask your instructor for the metal type of the unknowns. Record the metal of the unknowns in Calculations Table 1.
2. Place a clamp on the laboratory table and screw a threaded rod in the clamp. Slide the rod into the hole designed for that purpose in the base of the laboratory balance. Adjust the height of the balance above the table to allow the 1000 mL beaker to fit under the balance. Fill the beaker about three-fourths full of water.
3. Tie a piece of light string or thread around one of the unknowns and suspend it from one of the slots in the left arm underneath the balance. Determine the apparent mass with the unknown suspended completely below the surface of the water. Be sure that the unknown is not touching the side of the beaker or is in any way supported by anything other than the string. Record the mass reading of the balance as m_1 in Data Table 1.
4. Repeat Steps 1 through 3 for the second unknown.
5. From Appendix II obtain the known SG of the metal for each of the unknowns and record those values in Data Table 1.

Object Density Less than Water

1. Using the laboratory balance, determine the mass of the cork and record it as m in Data Table 2.
2. Determine the apparent mass with the cork suspended in air and a lead weight tied below submerged in water as shown in Figure 18-2(a). Record that value as m_1 in Data Table 2.

3. Determine the apparent mass with both the cork and the lead weight submerged in water as shown in Figure 18-2(b). Record that value as m_2 in Data Table 2.

Liquid Unknown

1. Use one of the unknown metal cylinders from the original procedure. Its mass m and its apparent mass in water have already been determined. Record those values in Data Table 3.
2. Use a very clean beaker and fill it about three-fourths full of alcohol. Determine the apparent mass of the metal cylinder in alcohol. Record it as m_L in Data Table 3. Return the alcohol to its container when you have finished with it. Be very careful not to have any spark or flame near the alcohol.
3. Repeat Steps 1 and 2 for the other unknown metal.
4. From Appendix II determine the known value of the specific gravity of alcohol and record it in Data Table 3.

CALCULATIONS

Density Greater than Water

1. Use Equation 7 to calculate the *SG* for each of the unknowns. Record the experimental values of the specific gravity in Calculations Table 1.
2. Calculate the percentage error in your value of the *SG* for each of the unknowns compared to the known values. Record them in Calculations Table 1.

Density Less than Water

1. Use Equation 8 to calculate the specific gravity *SG* for the cork and record it in Calculations Table 2.

Liquid Unknown

1. Use Equation 9 to determine the specific gravity of the alcohol for the two trials with the different metals. Record those values in Calculations Table 3.
2. Calculate the percentage error in each of the two measurements of the specific gravity of alcohol. Record the results in Calculations Table 3.