Name ______________________ Section ____________ Date ____________

Laboratory 41

Focal Length of Lenses

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. Mark the following statements about lenses as true or false.

_________a. Incident parallel light rays converge if the lens focal length is negative.

_________b. If the path of converging light rays are traced backwards, they appear to have come from a point called the "focal point."

_________c. A double convex lens has a negative focal length.

_________d. The focal length of a lens is always positive.

2. A double convex lens is made from glass whose index of refraction is $n = 1.50$. The *magnitudes* of its radii of curvature R_1 and R_2 are 10.0 cm and 15.0 cm, respectively. What is the focal length of the lens? Show your work.

$f =$ ________________ cm

3. What is a real image? What is a virtual image?

4. For a diverging lens, state what kinds of images can be formed and the conditions under which those images can be formed.

5. For a converging lens, state what kinds of images can be formed and the conditions under which those images can be formed.

6. A lens has a focal length of $f = +10.0$ cm. If an object is placed 30.0 cm from the lens, where is the image formed? Is the image real or virtual? Show your work.

7. An object is 16.0 cm from a lens. A real image is formed 24.0 cm from the lens. What is the focal length of the lens? Show your work.

8. One lens has a focal length of +15.0 cm. A second lens of focal length +20.0 cm is placed in contact with the first lens. What is the equivalent focal length of the combination of lenses? Show your work.

9. Two lenses are in contact. One of the lenses has a focal length of +10.0 cm when used alone. When the two are in combination, an object 20.0 cm away from the lenses forms a real image 40.0 cm away from the lenses. What is the focal length of the second lens? Show your work.

Laboratory 41

Focal Length of Lenses

OBJECTIVES

When a ray of light is incident upon the interface between two media in which the speed of light is different, the ray changes direction as it passes from one medium into the other. The process is called "refraction," and the different media are characterized by a constant called the "index of refraction." Lenses are devices that can cause parallel rays of light to converge or diverge by appropriate shaping of the interface between media of differing index of refraction. This laboratory will use an optical bench and several lenses either alone or in combination to accomplish the following objectives:

1. Demonstration that converging lenses form real images and diverging lenses form virtual images
2. Measurement of the focal length of converging lenses by forming a real image of a very distant object
3. Determination of the equivalent focal length of two lenses in contact in terms of their individual focal lengths
4. Measurement of the focal length of a diverging lens by using it in combination with a converging lens to form a real image

EQUIPMENT LIST

1. Optical bench with holders for lenses, a screen, and an object
2. Lamp with object painted on its face to serve as an illuminated object
3. Three lenses (focal lengths approximately +20, +10, and −30 cm)
4. Meter stick, screen on which to form image, masking tape

THEORY

When a beam of light rays parallel to the central axis of a lens is incident upon a converging lens, the rays are brought together at a point called the "focal point" of the lens. The distance from the center of the lens to the focal point is called the "focal length" of the lens, and it is a positive quantity for a converging lens.

When a parallel beam of light rays is incident upon a diverging lens the rays diverge as they leave the lens; however, if the path of the outgoing rays are traced backward, they appear to have emerged from a point called the "focal point" of the lens. The distance from the center of the lens to the focal point is called the "focal length" of the lens, and it is a negative quantity for a diverging lens.

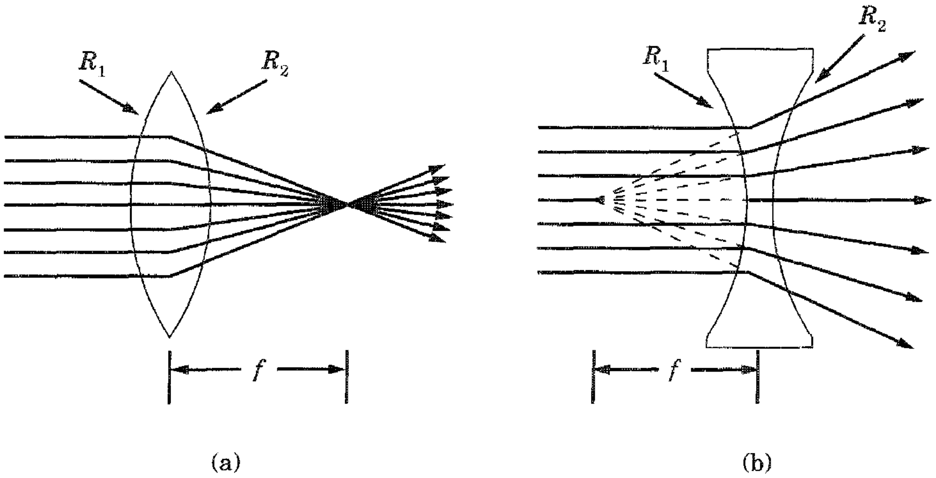

Figure 41.1 Ray diagram for converging and diverging showing the definition of the focal length for both the converging and diverging case.

In Figure 41.1, two common types of lenses are pictured. The first, shown in part (a), is a double convex lens whose focal length is positive, and thus it brings the incident parallel rays to a focus. The second type of lens, shown in part (b), is a double concave lens whose focal length is negative, and thus it causes the incident parallel rays to diverge. They appear to come from a fixed point as shown. In general, a lens is converging or diverging depending on the curvature of its surfaces. In Figure 41.1 the radii of curvature of the surfaces of the two lenses are denoted as R_1 and R_2. The general relationship that determines the focal length f in terms of the radii of curvature and the index of refraction n of the glass from which the lens is made is called the "lens makers' equation." It is given by

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{1}$$

For the converging lens shown in Figure 41.1(a), the radius R_1 is positive and the radius R_2 is negative, but for the diverging lens of part (b), the radius R_1 is negative and the radius R_2 is positive. The signs of these radii are determined according to a sign convention that is described in most elementary textbooks.

As an example, consider a double convex lens like the one shown in Figure 41.1(a) made from glass of index of refraction 1.60 with radii of curvature R_1 and R_2 of *magnitude* 20.0 and 30.0 cm, respectively. According to the sign convention given above, that would mean $R_1 = +20.0$ cm and $R_2 = -30.0$ cm. Putting those values into equation 1 gives a value for the focal length f of +20.0 cm.

Essentially, equation 1 states that a lens that is thicker in the middle than at the edges is converging, and a lens that is thinner in the middle than at the edges is diverging. Therefore, a lens can be classified as converging or diverging merely by taking it between one's fingers to see if the lens is thicker at its center than at its edge.

Lenses are used to form images of objects. There are two possible kinds of images. The first type, called a "real image," is one that can be focused on a screen. For a real image, light actually passes through the points at which the image is formed. The second type of image is called a "virtual image." For a virtual image, light does not actually pass through the points at which the image is formed, and the image cannot be focused on a screen. Diverging lenses can form only virtual images, but converging lenses can form either real images or virtual images. Whether the image formed by a converging lens is real or virtual depends on how far the object is from the lens compared to the focal length of the lens. If an object is further from a converging lens than its focal length, a real image is formed. However, if the object is closer to a converging lens than the focal length, the image formed is virtual. Note that whenever a virtual image is formed it will serve as the object for some other lens system to ultimately form a real image. Often the other lens system is the human eye, and the real image is formed on the retina of the eye.

In the process of image formation, the distance from an object to the lens is called the "object distance," or p, and the distance of the image from the lens is called the "image distance," or q.

The relationship between the object distance p, the image distance q, and the focal length of the lens f is given by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{2}$$

Note that equation 2 is valid both for converging (positive f) and for diverging (negative f) lenses. Normally, the object distance is considered positive. In that case, a positive value for the image distance means that the image is on the opposite side of the lens from the object, and the image is real. A negative value for the image distance means that the image is on the same side of the lens as the object, and the image is virtual.

When a lens is used to form an image of a very distant object, the image distance p is very large. For that case, the term $1/p$ in equation 2 is very small and is thus negligible compared to the other terms $1/q$ and $1/f$ in that equation. Therefore, for the case of a very distant object equation 2 becomes

$$\frac{1}{q} = \frac{1}{f} \tag{3}$$

For this case, the image distance is equal to the focal length. This provides a quick and accurate way to determine the focal length of a converging lens. The method is only applicable to a converging lens because the image must be focused on a screen. Since a diverging lens cannot form a real image, this technique will not work directly for a diverging lens.

If two lenses with focal lengths of f_1 and f_2 are placed in contact, the combination of the two act as a single lens of effective focal length f_e. The effective focal length of the two lenses in contact f_e is related to the individual focal lengths of the lenses f_1 and f_2 by

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} \tag{4}$$

Equation 4 holds for any combination of converging and diverging lenses. If the individual lenses f_1 and f_2 are converging, then the effective focal length f_e will, of course, also be converging. If one of the lenses is converging and the other is diverging, then the effective focal length can be either converging or diverging depending on the values of f_1 and f_2. If the converging lens has a smaller magnitude than the diverging lens, then the effective focal length will be converging. This fact can be used to determine the focal length of an unknown diverging lens if it is used in combination with a converging lens whose focal length is short enough to produce a converging combination.

EXPERIMENTAL PROCEDURE—FOCAL LENGTH OF A SINGLE LENS

1. Place one of the three lenses in a lens holder on the optical bench and place the screen in its holder on the optical bench (Figure 41.2). Place the optical bench in front of a window in the laboratory and point the bench toward some distant object. Adjust the distance from the lens to the screen until a sharp real image of the distant object is formed on the screen. You will be able to form such an image for only two of the three lenses. This experimental arrangement satisfies the conditions of equation 3. Therefore, the measured image distance is equal to the focal length of the lens. Record these measured image distances in Data Table 1 as the focal length of the two lenses for which the method works. Call the lens with the longest focal length A, the one with the shortest focal length B, and the one for which no image can be formed C.

2. Place lens B in the lens holder on the optical bench and use the lamp with the object painted on its face as an object. For various distances p of the object from the lens, move the screen until a sharp real image is formed on the screen. For each value of p measure the image distance q from the screen to the lens. Make sure that the lens, the object, and the screen are at the center of their respective holders. Try values for p of 20, 30, 40, and 50 cm determining the value of q for each case. If these values of p do not work for your lens, try other values until you find four values that differ by at least 5 cm. Record the values for p and q in Data Table 2.

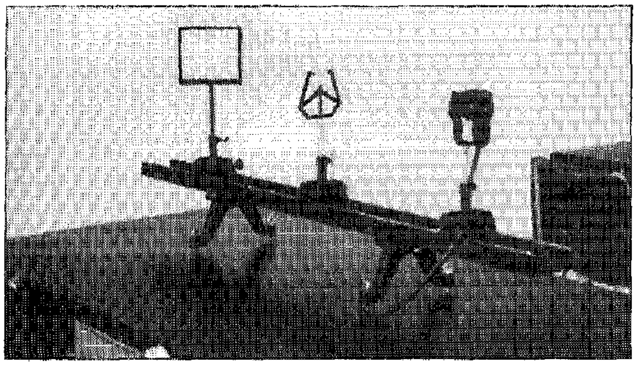

Figure 41.2 Optical bench with object, lens, and screen on which a real image is formed.

CALCULATIONS—FOCAL LENGTH OF A SINGLE LENS

1. Using equation 2, calculate the values of the focal length f for each of the four pairs of object and image distances p and q. Record them in Calculations Table 2.

2. Compute the mean $\bar{f}$ and the standard error $\alpha_{\bar{f}}$ for the four values for the focal length f and record them in Calculations Table 2.

3. The mean $\bar{f}$ represents the measurement of the focal length of lens B using finite object distances. Compute the percentage difference between $\bar{f}$ and the value determined using essentially infinite object distance in Data Table 1. Record the percentage difference between the two measurements in Calculations Table 2.

EXPERIMENTAL PROCEDURE—FOCAL LENGTH OF LENSES IN COMBINATION

1. Place lens A and lens B in contact, using masking tape to hold the edges of the two lenses parallel. Measure the focal length of the combination f_{AB} both by the very distant object method and by the finite object method. For the finite object method, use just one value of the object distance p and determine the image distance q. Record the results for both methods in Data Table 3.
2. Place lens B and lens C in contact, using masking tape to hold the edges of the two lenses parallel. Repeat the measurements described in step 1 above for these lenses in combination. Record the results in Data Table 3.

CALCULATIONS—FOCAL LENGTH OF LENSES IN COMBINATION

1. From the data for lenses A and B in Data Table 3 calculate the value of f_{AB} from the value of p and q using equation 2. Record that value of f_{AB} in Calculations Table 3.
2. Record the value of f_{AB} determined by the very distant object method in Calculations Table 3.
3. Calculate the average $\overline{f_{AB}}$ of the two values for f_{AB} determined in steps 1 and 2. This average value $\overline{f_{AB}}$ will be considered to be the experimental value for the combination of these two lenses.
4. Using equation 4, calculate a theoretical value expected for the combination of lenses A and B. Use the values determined in Data Table 1 by the distant object method for the values of f_A and f_B in the calculation. Record this value as $(f_{AB})_{\text{theo}}$ in Calculations Table 3.
5. Calculate the percentage difference between the experimental value and the theoretical value for f_{AB}. Record it in Calculations Table 3.
6. From the data for lenses B and C in Data Table 3, calculate the value of f_{BC} from the values of p and q using equation 2. Record that value of f_{BC} in Calculations Table 3.
7. Record the value of f_{BC} determined by the very distant object method in Calculations Table 3.
8. Calculate the average $\overline{f_{BC}}$ of the two values for f_{BC} determined in steps 6 and 7. This average value $\overline{f_{BC}}$ will be considered to be the experimental value for the combination of these two lenses.
9. Using the value of $\overline{f_{BC}}$ determined in step 8 and the value of f_B from Data Table 1 for the focal length of B, calculate the value of f_C, the focal length of lens C using equation 4. Record the value of f_C in Calculations Table 3.

Name ______________________ Section ____________ Date ____________

Laboratory 41

Focal Length of Lenses

LABORATORY REPORT

Data Table 1

Lens	Image Distance	Focal Length
A		$f_A =$
B		$f_B =$

Data Table 2

p (cm)	q (cm)

Calculations Table 2

f_B (cm)	$\overline{f_B}$	α_f	% difference

SAMPLE CALCULATIONS

Data Table 3

Lenses	$q = f$ (infinite object)	p	q
A and B	$f_{AB} =$		
B and C	$f_{BC} =$		

Calculations Table 3

Lenses A and B	
f_{AB} (infinite object)	
f_{AB} (from p and q)	
$\overline{f_{AB}}$	
$(f_{AB})_{\text{theo}}$	
% difference	

Lenses B and C	
f_{BC} (infinite object)	
f_{BC} (from p and q)	
$\overline{f_{BC}}$	
f_C	

SAMPLE CALCULATIONS

QUESTIONS

1. Why is it not possible to form a real image with lens C alone?

2. Take lens C between your thumb and index finger. Is it thinner at the center of the lens or thicker? Take lens B between your thumb and index finger. Is it thinner at the center of the lens or thicker? From this information alone, what can you conclude about lenses C and B?

3. Consider the percentage difference between the two measurements of the focal length of lens B. Express α_f as a percentage of $\overline{f}_B$. Is the percentage difference between the two measurements less than the percentage standard error?

4. Compare the agreement between the experimental and theoretical values of f_{AB} the focal length of lenses A and B combined. Does this data suggest that equation 4 is a valid model for the equivalent focal length of two lenses in contact?

5. If lens A and lens C were used in contact, could they produce a real image? State clearly the basis for your answer.