Name ______________________ Section ____________ Date ____________

Laboratory 37

Alternating Current RC and LCR Circuits

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. In a series RC circuit such as the one in Figure 37.1, the following phase relationship exists between the generator voltage V, the capacitor voltage V_C, and the resistor voltage V_R. (a) V and V_R are in phase, V_C lags V_R by 90°; (b) V_C and V_R are at angle ϕ, V leads V_C by 90°; (c) V_C lags V_R by 90°, V lags V_R by angle ϕ; or (d) V_R lags V_C by ϕ, V_C leads V by 90°.

2. If a series RC circuit has $V_R = 12.6$ V and $V_C = 10.7$ V, the generator voltage must be (a) 12.6 V, (b) 23.3 V, (c) 1.9 V, or (d) 16.5 V. Show your work.

3. A series RC circuit has $\omega = 2000$ rad/s and $R = 300\ \Omega$. The voltage V_C is measured to be 4.76 V, and V_R is measured to be 6.78 V. What is the value of C? Show your work. (a) 2.37 μF, (b) 5.00 μF, (c) 1.17 μF, or (d) 4.67 μF.

4. A series RC circuit of $R = 500\ \Omega$ and $C = 3.00\ \mu$F is measured to have $V_R = 8.07$ V and $V_C = 6.68$ V. What is the current I, and what is the value of the angular frequency ω? Show your work.

   I = ______________ A ω = ______________ rad/s

5. A series LCR circuit consists of an inductor of inductance L and resistance r, a capacitor C, a resistor R, and a generator of voltage V. Mark as true or false the following statements concerning the relative phase of V, V_L, V_r, V_C, and V_R.

_________ 1. V_r and V_R are in phase.

_________ 2. V_L leads V_C by 90°.

_________ 3. V is at angle ϕ relative to V_R.

_________ 4. $V_L - V_C$ is in phase with V_r.

_________ 5. V_C lags V_R by 90°.

6. Measurements on the circuit described in question 5 give $V_L = 10.76$ V, $V_C = 5.68$ V, $V_R = 6.32$ V, and $V_r = 3.75$ V. What is the generator voltage V? Show your work.

V = ________________ V

7. An inductor with $L = 150$ mH and $r = 200\ \Omega$ is in series with a capacitor, a resistor, and a generator of $\omega = 1000$ rad/s. The voltage across the inductor V_{ind} is measured to be 10.87 V, V_R is measured to be 4.65 V, and V_C is measured to be 5.96 V. What is the generator voltage V? (*Hint*: This is the measurement to be performed in this laboratory for LCR circuits. Use equations 7 and 8 to find V_L and V_r, and then use equation 4 to find V.) Show your work.

Laboratory 37

Alternating-Current RC and LCR Circuits

This laboratory assumes that Laboratory 36 has previously been performed, and that the measurements made of the inductance and resistance of an inductor coil have been recorded and retained for use in this laboratory. That same inductor is to be used in the *LCR* circuit, which is to be constructed in this laboratory.

OBJECTIVES

If a sinusoidally varying source of emf with a frequency f is placed in series with a resistor and a capacitor, the current I varies with time but is the same in each element of the circuit at any given instant. The current I will vary with the same frequency f as the generator, but it will be shifted in phase by an angle ϕ relative to the generator. The voltage across each of the circuit elements has its own characteristic phase relationship with the current. The resistor voltage V_R is in phase with the current I, the capacitor voltage V_C lags the current by a phase angle of 90°, and the generator voltage V lags the current by the angle ϕ whose value is dependent on the circuit parameters.

If an inductor of inductance L and resistance r is now added to the above circuit to form a series *LCR* circuit, the phase relationship for the resistor and capacitor are the same, and the inductor voltage has two components. One component is the voltage V_r across the resistance r, and it is in phase with the current. The other component V_L is the voltage across the inductance L, and it leads the current by 90°.

Measurements of the voltage across each element in a series *RC* circuit and of the voltage across each element in a series *LCR* circuit will be used to accomplish the following objectives:

1. Demonstration that the voltage across the resistor V_R and the voltage across the capacitor V_C are 90° out of phase in an *RC* circuit
2. Determination of the value of the capacitance of a capacitor in an *RC* circuit
3. Verification of the phase relationships among the voltages across the resistor, the capacitor, and the inductor in an *LCR* circuit

EQUIPMENT LIST

1. Sine-wave generator (variable frequency, 5 V peak-to-peak amplitude)
2. Resistance box
3. A 100-mH inductor (same one used in Laboratory 36 whose value of L and r have already been determined)
4. A 1.00-μF capacitor
5. AC voltmeter (digital readout, capable of measuring high frequency)
6. Compass and protractor

THEORY—RC CIRCUIT

Consider a series circuit consisting of a capacitor C, a resistor R, and a sine-wave generator of frequency f as shown in Figure 37.1. Also shown in the figure is a phasor diagram for the generator voltage V, the voltage across the resistor V_R, and the capacitor voltage V_C. It is assumed that all voltages discussed in this laboratory are root-mean-squared values. Since the voltages V_R and V_C are 90° out of phase, the voltages V, V_R, and V_C form a right triangle as shown in Figure 37.1. Therefore, the equation relating the magnitudes of the measured voltages in an RC circuit is

$$V = \sqrt{V^2{}_C + V^2{}_R} \quad (1)$$

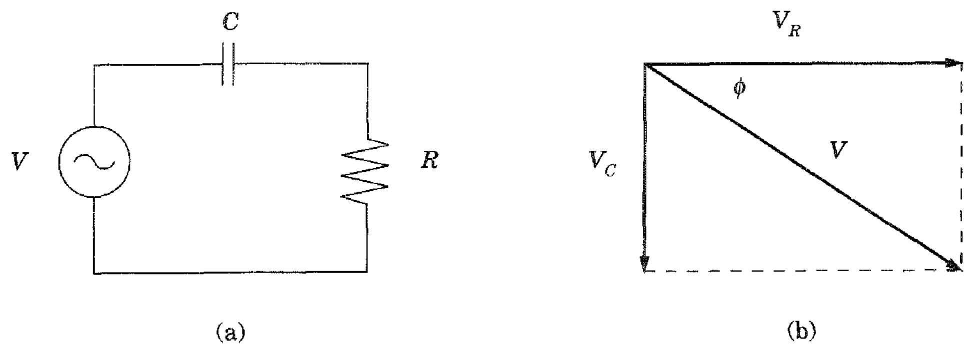


Figure 37.1 *RC* circuit and the phasor diagram for the voltage across each element.

Note that equation 1 is valid only for a pure capacitor with no resistive component. If measurements on a real capacitor show agreement with equation 1, it would indicate that the capacitor has no significant resistive component.

In an RC circuit, the current I is the same in each element of the circuit, and the relationships between the voltage and the current for the resistor and capacitor are

$$V_R = IR \quad \text{and} \quad V_C = I\left(\frac{1}{\omega C}\right) \quad (2)$$

The quantity $1/\omega C$ is called the "capacitive reactance," and it has units of ohms. If the current is eliminated between the two equations in 2, an equation for C is given by

$$C = \left(\frac{1}{\omega R}\right)\left(\frac{V_R}{V_C}\right) \quad (3)$$

Thus, a value for the capacitance of an unknown capacitor can be determined from equation 3 if ω and R are known and V_R and V_C are measured.

EXPERIMENTAL PROCEDURE—RC CIRCUIT

1. Connect the capacitor provided in series with the sine-wave generator and a resistance box to form a circuit like that shown in Figure 37.1. Set the generator output to maximum voltage and set the frequency to 250 Hz. Record the value of f in Data Table 1. Set the resistance box to a value of 300 Ω and record that value as R in Data Table 1.

2. Using the AC voltage scale on the voltmeter, very carefully measure the generator voltage V, the capacitor voltage V_C, and the voltage across the resistor V_R. Record those values in Data Table 1.
3. Repeat the above procedure for three other cases using R = 500, 700, and 900 Ω. Do not assume that the generator voltage stays the same. Even though the voltage setting is left at the maximum setting, the generator output might change slightly in response to changes in R. Therefore, be sure to measure all three voltages for each value of R.
4. Obtain a value for the capacitance of your capacitor from your instructor. Record this known value as C_k in Data Table 1.

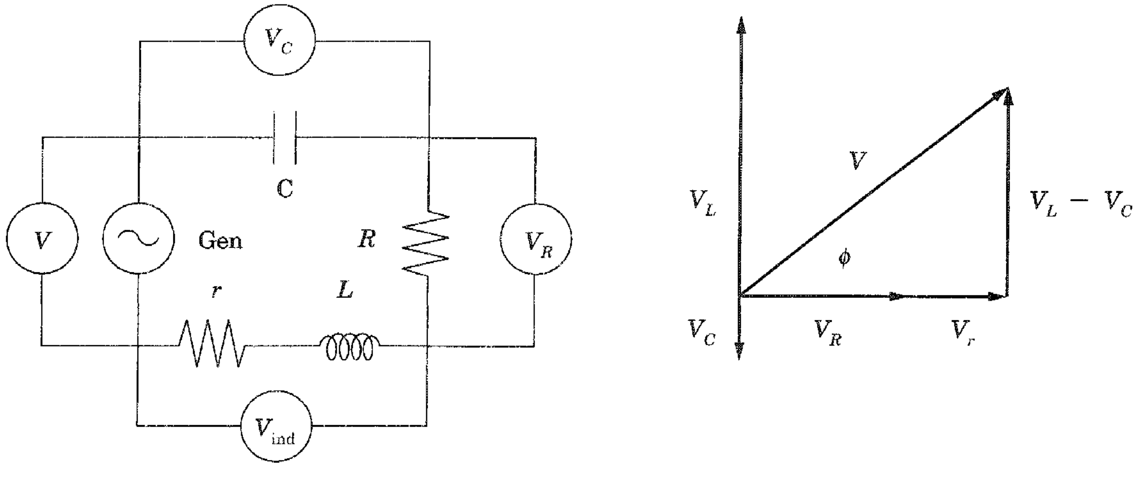
CALCULATIONS—*RC* CIRCUIT

1. From the known value of the frequency f, calculate the value of the angular frequency ω ($\omega = 2\pi f$). Record the value of ω in Calculations Table 1.
2. Calculate the quantity $\sqrt{V^2_C + V^2_R}$ for each case and record the values in Calculations Table 1.
3. Calculate the percentage error in the quantity $\sqrt{V^2_C + V^2_R}$ compared to each of the measured values of the generator voltage V. Record these percentage errors in Calculations Table 1.
4. Using equation 3, calculate the values of C from the measured values of V_R and V_C for each value of R. Record each value of C in Calculations Table 1. Calculate the mean $\bar{C}$ and the standard error α_C for the four values of C and record them in Calculations Table 1.
5. Calculate the percentage error in the value of $\bar{C}$ compared to the known value of the capacitance C_k.

THEORY—*LCR* CIRCUIT

Consider a series LCR circuit shown in Figure 37.2, with a generator of voltage V, a resistor R, a capacitor C, and an inductor having inductance L and resistance r. Note that the capacitor is assumed to have no resistance. Also shown in Figure 37.2 is the phasor diagram for the voltages V, V_R, V_C, V_L, and V_r. The figure shows that V_L and V_C are 180° out of phase, and V_R and V_r are in phase. Therefore, the quantities $V_L - V_C$, V, and $V_R + V_r$ form a right triangle, and thus they obey the following relationship:

$$V = \sqrt{(V_L - V_C)^2 + (V_R + V_r)^2} \quad \textbf{(4)}$$



(a) (b)

Figure 37.2 *LCR* circuit with voltmeter in the four positions to measure voltage across each element of the circuit. Also shown is a phasor diagram of all the relevant voltages.

It is not possible to measure either V_L or V_r directly. The only voltage associated with the inductor that can be experimentally measured is shown in Figure 37.2 as V_{ind}, and it is the vector sum of the voltages V_L and V_r. The relationship between V_{ind}, V_L, and V_r is shown in Figure 37.3. It is clear from that figure that the voltages V_{ind}, V_L, and V_r obey the following relationship:

$$V^2{}_{ind} = V^2{}_L + V^2{}_r \tag{5}$$

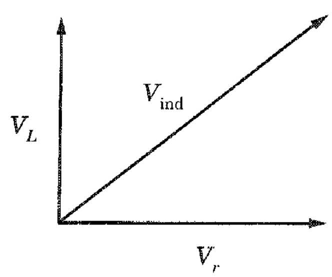


Figure 37.3 Phasor diagram for the voltages acrosss the components of the inductor.

Since the current I is the same in each element of the circuit, the voltages V_L and V_r can be expressed in terms of the current I as

$$V_L = I\,(\omega L) \quad \text{and} \quad V_r = Ir \tag{6}$$

The quantity ωL is called the "inductive reactance," and it has units of ohms. If equations 5 and 6 are combined, and the current I is eliminated it can be shown that

$$V_L = \frac{V_{ind}\ \omega L}{\sqrt{(\omega L)^2 + r^2}} \tag{7}$$

$$V_L = \frac{V_{ind}\ r}{\sqrt{(\omega L)^2 + r^2}} \tag{8}$$

Assuming that ω, L, and r are known, equations 7 and 8 can be used to determine V_L and V_r if V_{ind} is measured. These values of V_L and V_r combined with measured values of V_C and V_R can be used in equation 4 to verify the relationship between these quantities and the measured generator voltage V.

EXPERIMENTAL PROCEDURE—*LCR* CIRCUIT

1. Construct a series LCR circuit like the one shown in Figure 37.2 using the same capacitor used previously, the inductor used in Laboratory 36 (whose L and r were determined in that laboratory), a resistance box, and the sine-wave generator. Record the values of L and r in Data Table 2.
2. Set the generator output to maximum voltage and set the frequency to 800 Hz. Set the resistance box to a value of 200 Ω. Record the values of f and R in Data Table 2.
3. Using the AC voltage scale on the voltmeter, very carefully measure the generator voltage V, the capacitor voltage V_C, the resistor voltage V_R, and the inductor voltage V_{ind}. Record these values in Data Table 2.
4. Repeat the procedure of steps 1 through 3 with the generator frequency set to $f = 600$ Hz and the resistance box set to $R = 200\ \Omega$.
5. Repeat the procedure of steps 1 through 3 two more times, once with $R = 300\ \Omega$ and $f = 600$ Hz, and again with $R = 300\ \Omega$ and $f = 800$ Hz.

CALCULATIONS—*LCR* CIRCUIT

1. From the known value of f for each case, calculate the angular frequency ω ($\omega = 2\pi f$) and record the values in Calculations Table 2.
2. Using equations 7 and 8, calculate the four values of V_L and V_r and record them in Calculations Table 2.
3. Calculate and record in Calculations Table 2 the four values of $V_L - V_C$ and $V_R + V_r$.
4. For each of the four cases calculate and record in Calculations Table 2 the value of the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$.
5. Calculate the percentage error in the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$ compared to the measured value of the generator voltage V. Record the values of those percentage errors in Calculations Table 2.

Name ______________________ Section __________ Date __________

Laboratory **37**

Alternating Current RC and LCR Circuits

LABORATORY REPORT

Data Table 1

f = ________ Hz	C_k = ________ F			
R (Ω)				
V_C (V)				
V_R (V)				
V (V)				

Calculations Table 1

ω = rad/s				
$\sqrt{V^2_R + V^2_C}$				
% error				
C (F)				
$\overline{C}$ =	α_C =	% Error $\overline{C}$ =		

Data Table 2

r = ________ Ω	L= ________ H			
f (Hz)				
R (Ω)				
V (V)				
V_{ind} (V)				
V_C (V)				
V_R (V)				

Calculations Table 2

ω (rad/s)				
V_L (V)				
V_r (V)				
$V_L - V_C$ (V)				
$V_R + V_r$ (V)				
$\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$				
% error				

SAMPLE CALCULATIONS

QUESTIONS

1. Comment on the agreement between the measured generator voltage V and the quantity $\sqrt{V^2{}_C + V^2{}_R}$ for the RC circuit data.

2. Do your results for question 1 confirm that the capacitor has no resistance? State specifically how the data either do or do not confirm this expectation.

3. Evaluate the precision of the measurements of the value of the capacitor in the RC circuit.

4. Considering the given value of C_k as the true value, comment on the accuracy of your measurements of the capacitance.

5. Comment on the agreement between the measured generator voltage V and the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$ in the LCR circuit.

6. Do your results confirm the phasor diagram of Figure 37.2 as a correct model for the addition of the voltages in an LCR circuit?