Name ______________________ Section ____________ Date ____________

Laboratory 34

Kirchhoff's Rules

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. Consider the circuit in Figure 34.1. Choose any of the following statements about the circuit that are true. More than one may be correct. (a) It is a single-loop circuit. (b) It is a multiloop circuit. (c) Assuming R_1 and R_2 were known, the currents could be determined, but only if Kirchhoff's rules were used. (d) Assuming R_1 and R_2 were known the currents could be determined without the use of Kirchhoff's rules.

2. In the circuit of Figure 34.1, if $I_1 = 2.00$ A and $I_2 = 0.75$ A, what is the value of I_3?

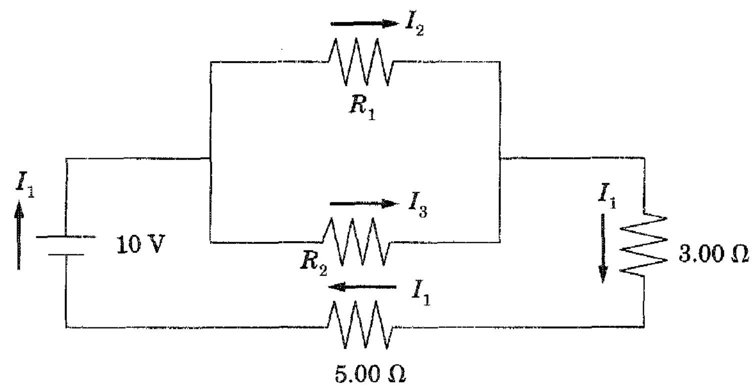

Figure 34.1 Circuit for questions 1 to 4.

For questions 3 and 4 assume that the value of R_1 and R_2 in Figure 34.1 are both 4.00 Ω.

3. What is the equivalent resistance of the circuit?

4. What is the current in the 3.00-Ω resistor?

5. Consider the circuit of Figure 34.2. Apply Kirchhoff's rules to the circuit and write three equations in terms of known circuit elements and the unknown currents shown in the figure.

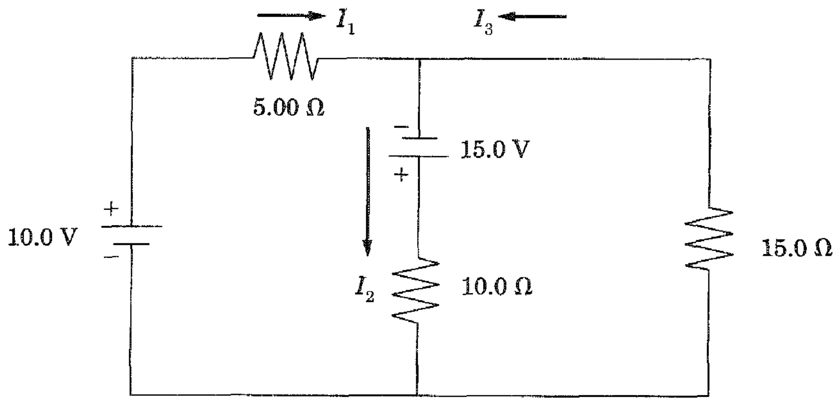

Figure 34-2 Multiloop circuit.

6. Solve the three equations that you wrote in question 5 for the values of the currents.

7. If any of the current values obtained in the solution to question 6 are negative, explain the significance of a negative value for a current.

Laboratory 34
Kirchhoff's Rules

OBJECTIVES

Multiloop circuits containing several sources of emf and several resistors will be constructed, and measurements of the voltage and current for each of the elements of the circuit will be used to achieve the following objectives:

1. Demonstration of the type of circuit to which Kirchhoff's rules must be applied
2. Theoretical application of Kirchhoff's rules to several circuits and solution for the expected currents in the circuit
3. Comparison between the theoretical values predicted by Kirchhoff's rules and the measured experimental values of the currents in multiloop circuits

EQUIPMENT LIST

1. DC voltmeter (preferably digital, 0–20 V)
2. DC ammeter (preferably digital, 0–1000 mA)
3. Two sources of emf (DC power supplies, up to 12 V)
4. Three or four resistors or resistor boxes (range, 500–1000 Ω)
5. Digital ohmmeter (one for the class)
6. Connecting wires

THEORY

Consider the circuit in Figure 34.3. The circuit is labeled with all of the currents. The 2-Ω resistor, 8-Ω resistor, and the 12-V power supply have current I_1, while the 6-Ω resistor has current I_2 and the 3-Ω resistor has current I_3. This circuit is called a single-loop circuit because it can be solved by noting that the 6-Ω resistor and the 3-Ω resistor are in parallel. This means that their equivalent resistance is 2 Ω. That equivalent 2-Ω resistance is then in series with the 12-V power supply and the other two resistors, reducing the circuit to a single-loop circuit. The total resistance of the circuit is 12 Ω, and this resistance is across the 12-V power supply. The current in the 12-Ω equivalent resistance is therefore $I_1 = 1$ A. Application of Ohm's law to the remaining part of the circuit gives $I_2 = 1/3$ A and $I_3 = 2/3$ A.

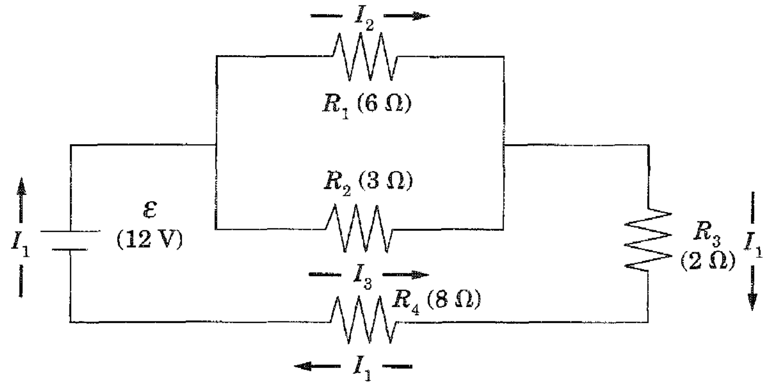

Figure 34.3 Single-loop circuit.

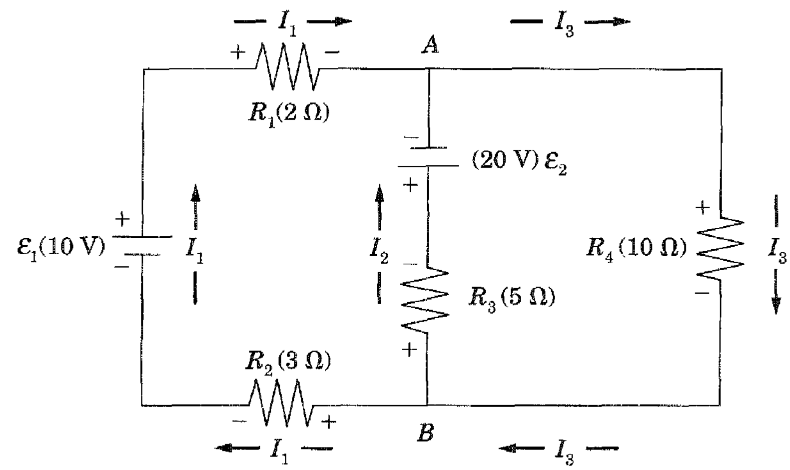

Figure 34.4 Multiloop circuit.

Consider now the circuit of Figure 34.4. This laboratory is concerned with the fundamental difference between circuits of the type depicted in Figure 34.3 and circuits of the type depicted in Figure 34.4. Note that the circuit in Figure 34.4 cannot be reduced to a single-loop circuit, but instead must be analyzed by other means. In the analysis of this circuit it is necessary to first define some terms. A point at which at least three possible current paths intersect is defined as a junction. For example, points A and B in Figure 34.4 are junctions. A closed loop is defined to be any path that starts at some point in a circuit and passes through elements of the circuit (in this case resistors and power supplies), and then arrives back at the same point without passing through any circuit element more than once. By this definition there are three loops in the circuit of Figure 34.4: (1) starting at B, going through the 10-V power supply to A, and then down through the 20-V power supply back to B; (2) starting at B, up through the 20-V power supply, and then around the outside through the 10-Ω resistor and back to B; and (3) completely around the outside part of the circuit. One can always traverse any loop in one of two directions, but regardless of which direction is chosen, the resulting equations are equivalent.

The solution for the currents in a multiloop circuit depends on analysis using two rules developed by a German professor named Gustav Robert Kirchhoff. The first of these rules is Kirchhoff's current rule (KCR) and can be stated in the following way:

KCR—The sum of currents into a junction = the sum of currents out of the junction

This rule actually amounts to a statement of conservation of charge. In effect it states that charge does not accumulate at any point in the circuit. The second rule is Kirchhoff's voltage rule (KVR) and can be stated as:

KVR—The algebraic sum of the voltage changes around any closed loop is zero.

This rule is essentially a statement of the conservation of energy, which recognizes that the energy provided by the power supplies is absorbed by the resistors.

In a multiloop circuit, the values of the resistors and the power supplies are known. As a first step it is necessary to determine how many independent currents are in the circuit, to label them, and then to assign a direction to each current. Application of Kirchhoff's rules to the circuits, treating the assigned currents as unknowns, will produce as many independent equations as there are unknown currents. Solution of those equations will determine the values of the currents.

In the application of KVR to a circuit, care must be taken to assign the proper sign to a voltage change across a particular element. The value of the voltage change across an emf $\mathcal{E}$ can be either $+\mathcal{E}$ or $-\mathcal{E}$ depending on which direction it is traversed in the loop. If the emf is traversed from the ($-$) terminal to the ($+$) terminal, the change in voltage is $+\mathcal{E}$. However, when going from the ($+$) terminal to the ($-$) terminal, the change in voltage is $-\mathcal{E}$. In the laboratory the terminal voltage of the sources of emf will be measured. It will be assumed that those values approximate the emf. In effect, this amounts to assuming that the source of emf has no internal resistance. This is a good approximation, but it is not strictly true, and thus will cause a small error.

Similarly, when a resistor R in which there is an assumed current I, is traversed in the loop in the same direction as the current, the voltage change is $-IR$. Conversely, if the resistor is traversed in the direction opposite that of the current, the voltage change is $+IR$. It is very important to understand that the sign of the voltage change across an emf is not affected by the direction of the current in the emf. The sign of the voltage change across a resistor, however, is completely determined by the current direction.

Consider the application of Kirchhoff's rules to the multiloop circuit of Figure 34.4. At the junction point A, currents I_1 and I_2 are going into the junction, and current I_3 is going out of the junction and KCR states that

$$I_1 + I_2 = I_3 \tag{1}$$

It might appear that applying KCR to the junction B would produce an additional useful equation, but in fact it would result in an equation that is equivalent to equation 1.

Applying KVR to the loop that starts at point B, goes through the 10-V power supply to A, and then down through the 20-V power supply back to B, gives the following equation.

$$-R_2 I_1 + \mathcal{E}_1 - R_1 I_1 + \mathcal{E}_2 + R_3 I_2 = 0 \tag{2}$$

Although normally the voltage change across a resistor is written as IR, it has been written here as RI in order to emphasize that I is the unknown, and R is essentially the coefficient of I. Replacing the symbols in equation 2 with the values of the resistances and emfs of the circuit gives

$$-3I_1 + 10 - 2I_1 + 20 + 5I_2 = 0 \tag{3}$$

The signs used in both equations 2 and 3 are consistent with the description given above for determining the signs of voltage changes. Furthermore, the (+) and (−) labeled on each side of the circuit elements of the circuit diagram are also consistent with that convention.

Finally, applying KVR to the loop that starts at B and goes clockwise around the right side of the circuit gives

$$-R_3 I_2 - \mathcal{E}_2 - R_4 I_3 = 0 \tag{4}$$

Substituting the values of the resistances and emf for the symbols in equation 4 results in

$$-5I_2 - 20 - 10I_3 = 0 \tag{5}$$

Equations 1, 3, and 5 are the three desired equations in the three unknowns I_1, I_2, and I_3. The solution of these equations gives values for the currents of $I_1 = 2.800$ A, $I_2 = -3.200$ A, and $I_3 = -0.400$ A. The fact that the currents I_2 and I_3 are negative is an indication that the original assumption of direction for these two currents was incorrect. The interpretation of the results of the solution is that there is a current of 2.800 A in the direction indicated in the figure for I_1, a current of 3.200 A in a direction opposite to that indicated in the figure for I_2, and a current of 0.400 A in a direction opposite to that indicated for I_3. This is a general feature of solutions for the currents in a multiloop circuit using Kirchhoff's rules. Even if the original assumption of the direction of a current is wrong, the solution of the equations leads to the correct understanding of the proper direction by virtue of the sign of the current.

EXPERIMENTAL PROCEDURE

1. Choose three resistors with the following values: $R_1 = 500\ \Omega$, $R_2 = 750\ \Omega$, and $R_3 = 1000\ \Omega$. If using resistance boxes these values can be chosen exactly, but if using standard resistors choose values as close as possible to the values listed. Using the ohmmeter measure the precise value of the resistors and record those values in Data Table 1.
2. Using two power supplies and the resistors R_1, R_2, and R_3, construct a circuit like that shown in Figure 34.5 with $\mathcal{E}_1 = 10$ V and $\mathcal{E}_2 = 5$ V. (*Caution*: If the laboratory is performed with batteries instead of power supplies make certain that no battery is subject to large reverse currents. Some batteries might be damaged or even explode if subjected to large reverse currents. This should be of minimal danger for the low currents that will result using voltages and resistors of the magnitude suggested.)

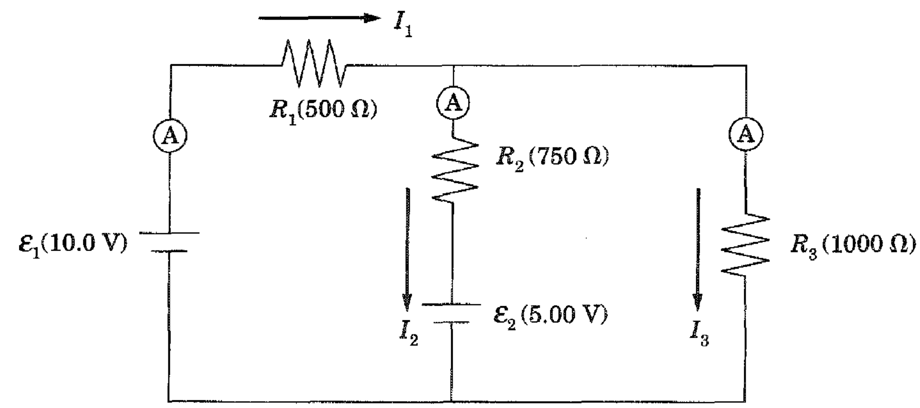

Figure 34.5 Experimental multiloop circuit with three unknown currents.

3. Measure the currents I_1, I_2, and I_3. Assuming that only one ammeter is available, the currents will have to be measured one at a time by placing the ammeter in the positions shown in the circuit diagram as a circle. Note that placing the ammeter in the circuit with the polarity shown in the circuit diagram will give positive readings when the current is in the direction assumed. If a modern digital ammeter is used for the measurements, the ammeter will give a positive reading if the current is in the direction assumed, and will give a negative reading if the current is in the opposite direction. If, however, an ammeter is used that properly deflects in only one direction, the meter could be damaged if the current is in the opposite direction from that assumed. In this case the polarity of the meter must be reversed. If there is any concern about damage to the ammeter, it may be prudent to perform the calculations (described in the next section) for the unknown currents and thus determine their magnitude and direction in advance.

4. Measure the values of the emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ with a voltmeter and record those values in Data Table 1. As previously noted, these values are really the terminal voltages of the power supplies which are assumed to approximate the emfs if the internal resistance of the sources are negligible.

5. Construct the circuit of Figure 34.6, which also has two power supplies but has four resistors. Choose values of $\mathcal{E}_1$ = 5 V, $\mathcal{E}_2$ = 10 V, R_1 = 1000 Ω, R_2 = 800 Ω, R_3 = 600 Ω, and R_4 = 500 Ω or values as close to those as possible. Following similar procedures to those described above, measure the values of the resistors and record those values in Data Table 2. Following similar procedures to those described above, measure each of the four currents and the emf of the power supplies and record those values in Data Table 2.

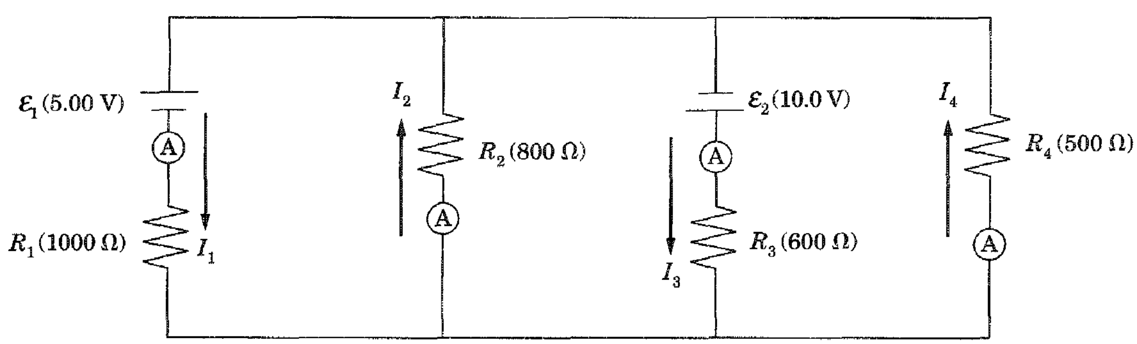

Figure 34-6 Experimental multiloop circuit with four unknown currents.

CALCULATIONS

1. Apply Kirchhoff's rules to the circuit of Figure 34.5 with the actual values that were used in your circuit. Three equations in the three currents I_1, I_2, and I_3 will result. One equation will be a KCR equation, and two will be KVR equations. Record those three equations in the appropriate place in Calculations Table 1.
2. Solve the three equations that you have written for the values of I_1, I_2, and I_3. Record those values in Calculations Table 1.
3. Calculate the percentage error of the experimental values of the current compared to the theoretical values for each of the currents. Record the results in Calculations Table 1.
4. Apply Kirchhoff's rules to the circuit of Figure 34.6 with the actual values that were used in your circuit. Four equations in the four currents I_1, I_2, I_3, and I_4 will result. One equation will be a KCR equation, and three will be KVR equations. Record those four equations in the appropriate place in Calculations Table 2.
5. Solve the four equations that you have written for the values of I_1, I_2, I_3 and I_4. Record those values in Calculations Table 2.
6. Calculate the percentage error of the experimental values of the current compared to the theoretical values for each of the currents. Record the results in Calculations Table 2.

Name ______________________ Section ____________ Date ____________

Laboratory 34
Kirchhoff's Rules

LABORATORY REPORT

Data Table 1

Power Supply Voltages $\mathcal{E}_1$ = ______ V $\mathcal{E}_2$ = ______ V	
Resistor Values (Ω)	Experimental Current (mA)
R_1 =	I_1 =
R_2 =	I_2 =
R_3 =	I_3 =

Calculations Table 1

Kirchoff's rules for the circuit (1) KCR — (2) KVR1 — (3) KVR2 —	
Theoretical Current (mA)	% Error of Experimental Current compared to the Theoretical Current
I_1 =	
I_2 =	
I_3 =	

Data Table 2

Power Supply Voltages $\mathcal{E}_1$ = ______ V $\mathcal{E}_2$ = ______ V	
Resistor Values (Ω)	Experimental Current (mA)
R_1 =	I_1 =
R_2 =	I_2 =
R_3 =	I_3 =
R_4 =	I_4 =

Calculations Table 2

Kirchoff's rules for the circuit (1) KCR — (2) KVR1 — (3) KVR2 — (4) KVR3 —	
Theoretical Current (mA)	% Error of Experimental Current compared to the Theoretical Current
I_1 =	
I_2 =	
I_3 =	
I_4 =	

SAMPLE CALCULATIONS

QUESTIONS

1. In Figure 34.5 what is the equation that relates the currents I_1, I_2, and I_3?

2. State quantitatively how well your experimental results for the circuit of Figure 34.5 agree with the expected relationship given in question 1. (*Hint*: Calculate the percentage difference between the experimental values of the two sides of the equation given in question 1.)

3. In Figure 34.6, what is the equation that relates the currents I_1, I_2, I_3, and I_4?

4. State quantitatively how well your experimental results for the circuit of Figure 34.6 agree with the expected relationship given in question 3.

5. Are the experimental values of the currents for the entire laboratory generally larger or smaller than the theoretical values expected for the currents?

6. It was pointed out in the laboratory that some error might be caused by neglect of the internal resistance of the emf. Would the internal resistance cause an error in the direction shown in your answer to question 5? State your reasoning for the direction of any error caused by the internal resistance.

7. An ideal ammeter has zero resistance. Real ammeters have small but finite resistance. Would ammeter resistance cause an error in the proper direction to account for the direction of your error indicated in question 5? State your reasoning.

8. The connecting wires in the experiment are assumed to have no resistance, but in fact have a finite resistance. Would this error be in the proper direction to account for the direction of the error stated in your answer to question 5? State your reasoning.