

Laboratory 3

Force Table and Vector Addition of Forces

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. Scalars are physical quantities that can be completely specified by their _____.
2. A vector quantity is one that has both _____ and _____.
3. Classify each of the following physical quantities as vectors or scalars:
 - (a) Volume _____
 - (b) Force _____
 - (c) Density _____
 - (d) Velocity _____
 - (e) Acceleration _____

Answer questions 4 to 7 with reference to Figure 3.1.

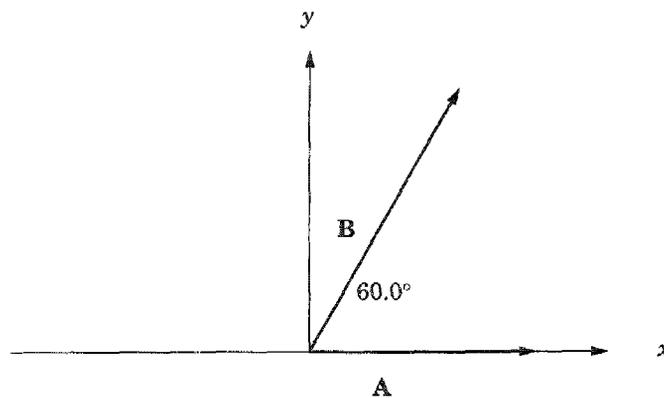


Figure 3.1 Addition of two force vectors.

4. If **A** stands for a force vector of magnitude 30.0 N and **B** stands for a force vector of magnitude 40.0 N acting in the directions shown in the figure, what is the magnitude and direction of the *resultant* obtained by the addition of these two vectors using the analytical method?

Magnitude = _____ N Direction (relative to x -axis) = _____ degrees

5. What is the *equilibrant* force that would be needed to compensate for the resultant force of the vectors **A** and **B**?

Magnitude = _____ N Direction (relative to x -axis) = _____ degrees

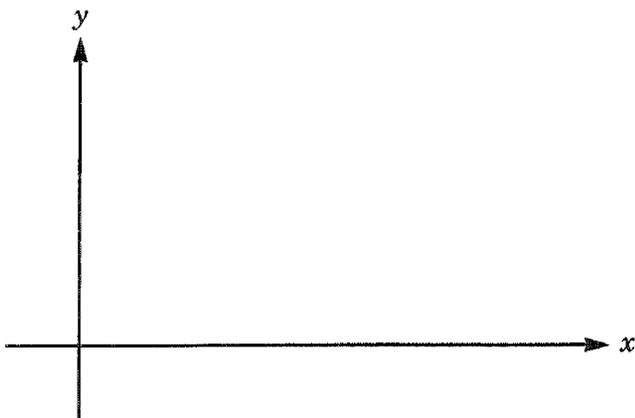
6. Figure 3.1 has been constructed to scale with 1.00 cm = 10.0 N. Using the graphical method, construct (on Figure 3.1) the resultant vector for the addition of **A** and **B** by the parallelogram method. Measure the length of the resultant vector and record it below. State the force represented by this length. Using a protractor, measure the angle that the resultant makes with the x -axis.

Resultant vector length = _____ cm

Force represented by this length = _____ N

Direction of resultant relative to x -axis = _____ degrees

7. Construct on the axes below a graphical solution to the problem in Figure 3.1 using the polygon method of vector addition. Use the scale 1.00 cm = 10.0 N.



Resultant vector length = _____ cm

Force represented by this length = _____ N

Direction of resultant relative to x -axis = _____ degrees

Force Table and Vector Addition of Forces**OBJECTIVES**

Physical quantities that can be completely specified by magnitude only are called "scalars." Examples of scalars include temperature, volume, mass, and time intervals. Some physical quantities have both magnitude and direction, and these are called "vectors." Examples of vector quantities include spatial displacement, velocity, and force. In this laboratory, a force table will be used to determine the magnitude and direction of several simultaneously applied forces to accomplish the following objectives:

1. Demonstration of the process of the addition of several vectors to form a resultant vector
2. Demonstration of the relationship between the resultant of several vectors and the equilibrant of those vectors
3. Illustration and practice of graphical solutions for the addition of vectors
4. Illustration and practice of analytical solutions for the addition of vectors

EQUIPMENT LIST

1. Force table with pulleys, ring, and string
2. Mass holders and slotted masses
3. Protractor and compass

THEORY

As an example of the process of vector addition, consider the case of several forces with different magnitudes and directions that act at the same point. It is desired to find the net effect produced by the several forces by finding a single force which is equivalent in its effect to the effect produced by the several applied forces. That single vector is called the resultant vector of the several applied vectors. This resultant vector can be found theoretically by a special addition process known as vector addition.

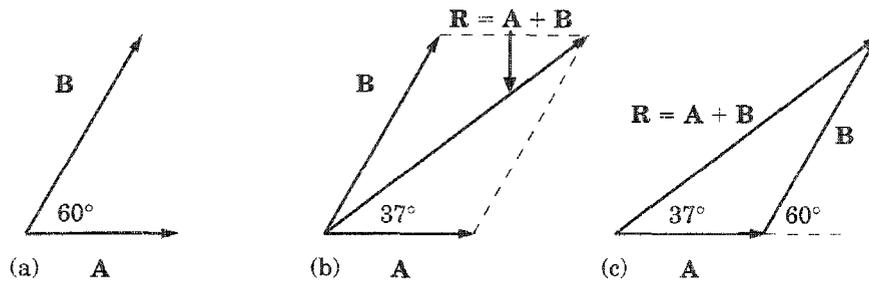


Figure 3.2 Illustration of the parallelogram and triangle addition of vectors.

First, consider the process of vector addition by graphical techniques. Figure 3.2(a) shows the case of two vectors **A** and **B**, which are assumed to represent two forces, with **A** having a magnitude of 20.0 N and **B** having a magnitude of 30.0 N. The scale of 1.00 cm = 10.0 N has been chosen; thus, these vectors are shown as 2.00 cm and 3.00 cm in length, respectively. The vectors are assumed to act at the same point, but 60.0° different in direction. Thus, the tails of the arrows are drawn from the same point, but the heads of the arrows point in directions 60.0° apart. Figure 3.2(b) shows the graphical addition process called the “parallelogram method.” The dotted lines shown were constructed with one parallel to **A** and equal in length to **A** drawn from the tip of **B**, and the other parallel to **B** and equal in length to **B** drawn from the tip of **A**. The resultant **R** of the vector addition of **A** and **B** was found by constructing the straight line from the point at the tails of the two vectors to the opposite corner of the parallelogram formed by the original vectors and the constructed dotted lines. A careful measurement of the length of **R** in Figure 3.2(b) shows it to be about 4.35 cm in length, and a measurement of the angle between **R** and **A** shows it to be about 37°. Since the scale is 1.00 cm = 10.0 N, the value of the resultant **R** is 43.5 N, and it acts in a direction 37° with respect to the direction of **A**.

In the graphical vector addition process known as the polygon method, one of the vectors is first drawn to scale. Then each successive vector to be added is drawn with its tail starting at the head of the preceding vector. The resultant vector is then the vector drawn from the tail of the first arrow to the head of the last arrow. Figure 3.2(c) shows this process for the case of only two vectors (for which the polygon method is the triangle method). Note that the second vector, **B**, must be drawn at the proper angle relative to **A** by extending a dotted line in the direction of **A** and constructing **B** relative to that dotted line. In Figure 3.2(c) the length of **R** is again seen to be 4.35 cm, corresponding to 43.5 N, and it acts at 37° with respect to **A**.

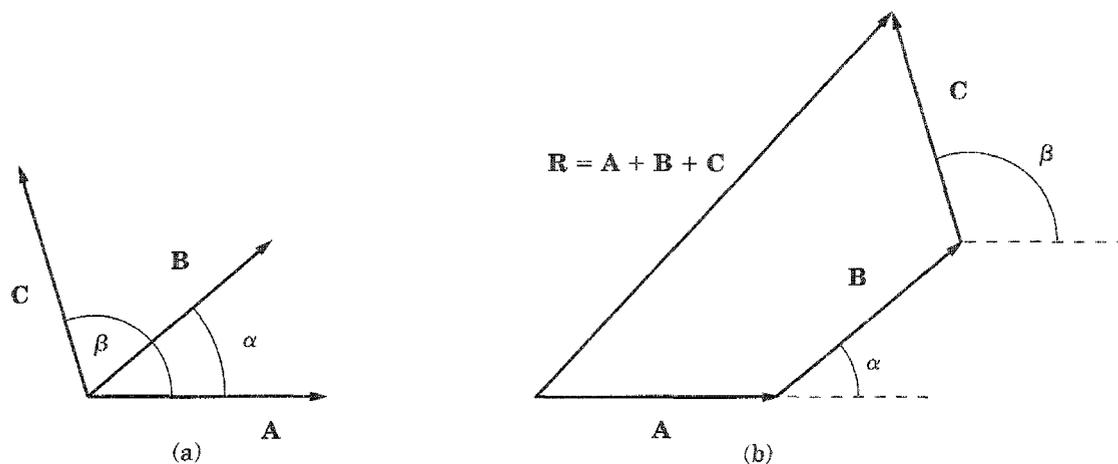


Figure 3.3 Illustration of the polygon method for vector addition.

The true polygon method is illustrated for the case of three vectors in Figure 3.3. The extension of the idea to four or more vectors should be clear. Note carefully that each successive vector must be drawn in the proper relationship to the previous vector. Thus, vector **A** is first drawn, **B** is then drawn at the proper angle α relative to **A**, and **C** is then drawn at the proper angle β relative to **B**. Finally, the resultant **R** is the vector connecting the tail of **A** and the head of **C**.

In each of the graphical methods, the results are exact in principle. In practice they depend greatly on the size of the scale chosen and the care with which the lengths are measured. In general, the larger the scale used, the better the accuracy of the results. In contrast, the analytical process is as exact as the number of significant figures chosen for the calculations. It uses trigonometry to express each vector in terms of its components projected on the axes of a rectangular coordinate system. The process of determining the components of a vector is illustrated in Figure 3.4. In Figure 3.4(a) is shown a vector **A**.

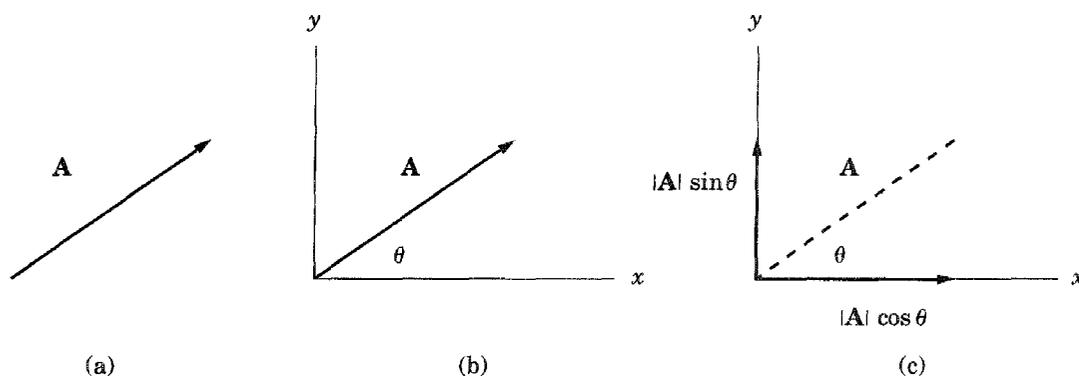


Figure 3.4 Illustration of analytical resolution of a vector.

In Figure 3.4(b) the vector **A** is shown with a rectangular coordinate system superimposed, and in Figure 3.4(c) the components of the vector along the axes are shown to be $|A| \cos \theta$ along the x -axis and $|A| \sin \theta$ along the y -axis.

In the process of vector addition, each vector to be added is first resolved into components as shown in Figure 3.4. The components along each axis are then added algebraically to produce the net components of the resultant vector along each axis. Those components are at right angles, and thus the magnitude of the resultant can be found from the Pythagorean theorem. Consider the case of three vectors **A**, **B**, and **C** shown in Figure 3.5. Assume that their magnitudes are 10.0, 15.0, and 12.0, and their directions relative to the x -axis are as shown in that figure. Taking the algebraic sum of each of the components of the three vectors leads to the following:

$$R_x = A_x + B_x + C_x = 10.0 \cos(0^\circ) + 15.0 \cos(30.0^\circ) - 12.0 \cos(45.0^\circ) = 14.5 \quad (1)$$

$$R_y = A_y + B_y + C_y = 10.0 \sin(0^\circ) + 15.0 \sin(30.0^\circ) + 12.0 \sin(45.0^\circ) = 16.0 \quad (2)$$

Note very carefully that the x -component of the **C** vector is negative because it points to the left, which is the negative x -direction. The magnitude of the resultant, R , is found to be the following because the components R_x and R_y are at right angles.

$$|R| = R = \sqrt{R_x^2 + R_y^2} = \sqrt{(14.5)^2 + (16.0)^2} = 21.6 \quad (3)$$

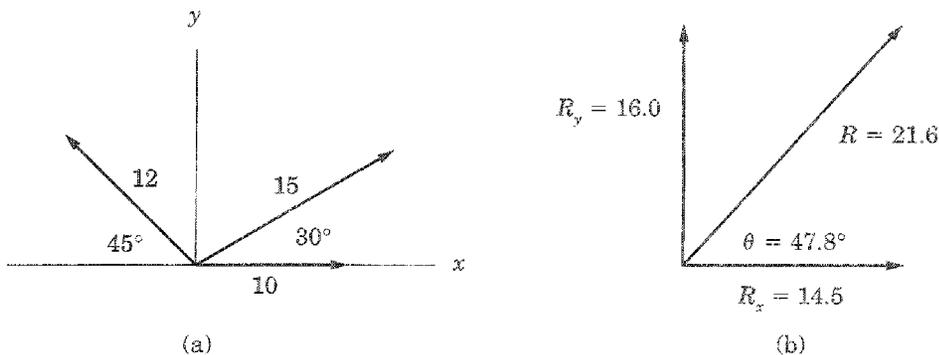


Figure 3.5 Illustration of the analytical addition of vectors.

Furthermore the angle θ that the resultant makes with the x -axis is given by the following:

$$\theta = \arctan \left(\frac{R_y}{R_x} \right) = \arctan \left(\frac{16.0}{14.5} \right) = 47.8^\circ \quad (4)$$

The process of experimentally determining the value of the resultant of several forces is complicated by the fact that when a nonzero resultant force acts on an object it tends to be accelerated. Thus, another force must be applied to produce an equilibrium. For example, if two known forces, F_1 and F_2 , are applied to some object, they will have some resultant F_R . In order to keep the object in equilibrium, a force equal in magnitude and opposite in direction of F_R must be applied. The force applied in order to produce equilibrium is called the "equilibrant force," or F_E .

For any number of applied forces (F_1, F_2, F_3, \dots) the equilibrant force F_E is the force that must be applied (along with the original applied forces) in order to keep an object in equilibrium. The magnitude and direction of F_E can be found by trial and error experimentally. The resultant force, F_R , can be found from the knowledge that F_R and F_E have the same magnitude but opposite directions.

The force table (Figure 3.6) used in the experiment is designed to allow application of forces of any chosen magnitude at any chosen angle. The force is provided by the gravitational attraction on masses that are attached to a ring by a string passing over a pulley. Each force is applied over a separate pulley, and the pulley positions can be adjusted to any desired position around a circular plate. It is important to note that the force applied when a mass is hung on the string is equal to the weight of the mass. The weight (N) is equal to the product of the mass (kg) multiplied by the acceleration due to gravity (9.80 m/s^2).

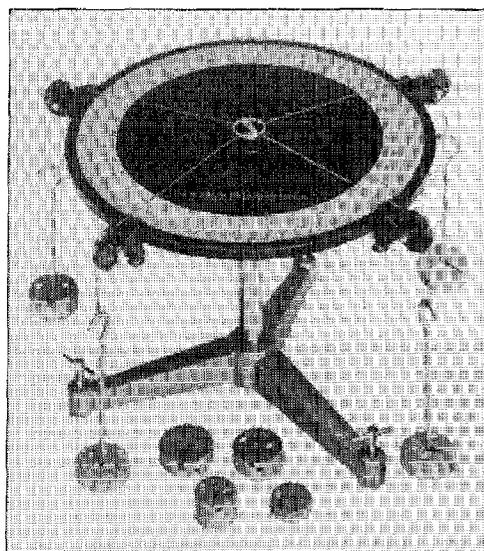


Figure 3.6 Force Table. (Photo courtesy of Sargent-Welch Scientific Co.)

EXPERIMENTAL PROCEDURE

Part 1: Two Applied Forces

1. Place a pulley at the 20.0° mark on the force table and place a total of 0.100 kg on the end of the string. Be sure to include the 0.050 kg of the mass holder in the total. Calculate the magnitude of the force (N) produced by the mass. Assume three significant figures for this and for all other calculations of force on the force table. Record the value of this force as F_1 in Data Table 1.
2. Place a second pulley at the 90.0° mark on the force table and place a total of 0.200 kg on the end of the string. Calculate the force produced and record as F_2 in Data Table 1.
3. Determine by trial and error the magnitude of mass needed and the angle at which it must be placed in order to place the ring in equilibrium. The ring is in equilibrium when it is centered on the force table. Jiggle the ring slightly to be sure that the equilibrium conditions are met. Be sure that all the strings are in such a position that they are directed along a line that passes through the center of the ring. This is crucial because all the forces must act through the point at the center of the table.
4. For the experimentally determined mass, calculate the force produced and record the magnitude and direction of this equilibrant force F_{E1} in Data Table 1.
5. From the value of the equilibrant force F_{E1} , determine the magnitude and direction of the resultant force F_{R1} and record them in Data Table 1.

Part 2: Three Applied Forces

1. Place a pulley at 30.0° with 0.150 kg on it, one at 100.0° with 0.200 kg on it, and one at 145.0° with 0.100 kg on it.
2. Calculate the force produced by those masses and record them as F_3 , F_4 , and F_5 in Data Table 2.
3. Following the procedure outlined steps 3 through 5 in Part 1 above, determine the equilibrant force and the resultant force and record their magnitudes and directions in Data Table 2 as F_{E2} and F_{R2} .

CALCULATIONS

Part 1: Two Applied Forces

1. Find the resultant of these two applied forces by scaled graphical construction using the parallelogram method. Using a ruler and protractor, construct vectors whose scaled length and direction represent F_1 and F_2 . For example, a convenient scale might be $1.00 \text{ cm} = 0.100 \text{ N}$. Be careful to note that all directions are given relative to the force table, and this must be taken into account in the graphical construction to ensure the proper angle of one vector to another. Read the magnitude and direction of the resultant from your graphical solution and record them in the appropriate section of Calculations Table 1.
2. Using trigonometry, calculate the components of F_1 and F_2 and record them in the analytical solution portion of Calculations Table 1. Add the components algebraically and determine the magnitude of the resultant by the Pythagorean theorem. Determine the angle of the resultant from the arc tan of the components. Record those results in Calculations Table 1.
3. Calculate the percentage error of the magnitude of the experimental value of F_R compared to the analytical solution for F_R . Also calculate the percentage error of the magnitude of the graphical solution for F_R compared to the analytical solution for F_R . For each of those comparisons, also calculate the magnitude of the difference in the angle. Record all values in Calculations Table 1.

Part 2: Three Applied Forces

1. Find the resultant of these three applied forces using scaled graphical construction, but for this case, use the polygon method. Read the magnitude and direction of the resultant from your graphical solution and record them in Calculations Table 2.
2. Using trigonometry calculate the components of all three forces, the components of the resultant, and, the magnitude and direction of the resultant. Record them all in Calculations Table 2.
3. Make the same error calculations for this problem as asked for in step 3 above in Part 1.

Laboratory 3**Force Table and Vector Addition of Forces****LABORATORY REPORT****Data Table 1**

Force	Mass (kg)	Force (N)	Direction
F_1	0.100		20.0°
F_2	0.200		90.0°
Equilibrant F_{E1}			
Resultant F_{R1}			

Data Table 2

Force	Mass (kg)	Force (N)	Direction
F_3	0.150		30.0°
F_4	0.200		100.0°
F_5	0.100		145.0°
Equilibrant F_{E2}			
Resultant F_{R2}			

SAMPLE CALCULATIONS

Calculations Table 1
Graphical Solution

Force	Mass (kg)	Force (N)	Direction
F_1	0.100		20.0°
F_2	0.200		90.0°
Resultant F_{R1}			

Analytical Solution

Force	Mass (kg)	Force (N)	Direction	x component	y component
F_1	0.100		20.0°		
F_2	0.200		90.0°		
Resultant F_{R1}					

Part 1 Error Calculations

Percent Error magnitude Experimental compared to Analytical = _____ %

Percent Error magnitude Graphical compared to Analytical = _____ %

Absolute Error in angle Experimental compared to Analytical = _____ degrees

Absolute Error in angle Graphical compared to Analytical = _____ degrees

Calculations Table 2
Graphical Solution

Force	Mass (kg)	Force (N)	Direction
F_3	0.150		30.0°
F_4	0.200		100.0°
F_5	0.100		145.0°
Resultant F_{R2}			

Analytical Solution

Force	Mass (kg)	Force (N)	Direction	x component	y component
F_3	0.150		30.0°		
F_4	0.200		100.0°		
F_5	0.100		145.0°		
Resultant F_{R2}					

Part 2 Error Calculations

Percent Error magnitude Experimental compared to Analytical = _____ %

Percent Error magnitude Graphical compared to Analytical = _____ %

Absolute Error in angle Experimental compared to Analytical = _____ degrees

Absolute Error in angle Graphical compared to Analytical = _____ degrees

QUESTIONS

1. Possible sources of error include: (1) friction in the pulleys, (2) the fact is ignored that the strings actually have mass, and (3) errors in direction of the forces if the strings do not act at 90° to a tangent to the rings. Based on the errors in your magnitude and direction, rank the relative importance of these errors for your data.
2. To determine the force acting on each mass it was assumed that $g = 9.80 \text{ m/s}^2$. The value of g at the particular place where the experiment is performed may be slightly different from that value. State clearly what effect (if any) that would have on the *percentage error* calculated for the comparisons. In order to test your answer to the question, leave g as a symbol in the calculation of the *percentage error*.
3. Two forces are applied to the ring of a force table, one at an angle of 20.0° , and the other at an angle of 80.0° . Regardless of the magnitudes of the forces, will the *equilibrant* be in the (a) first quadrant, (b) second quadrant, (c) third quadrant, (d) fourth quadrant, or (e) you cannot tell which quadrant from the available information?
4. Two forces, one of magnitude 2.00 N and the other of magnitude 3.00 N, are applied to the ring of a force table. The directions of both forces are unknown. Which *best* describes the limitations on R , the resultant?
(a) $R \leq 5 \text{ N}$ (b) $2 \text{ N} \leq R \leq 3 \text{ N}$ (c) $R \geq 3 \text{ N}$ (d) $1 \text{ N} \leq R \leq 5 \text{ N}$ (e) $R \leq 2 \text{ N}$.
5. Suppose the same masses are used for a force table experiment as were used in Part 1, but each pulley is moved 180° so that the 0.100-kg mass acts at 200° , and the 0.200-kg mass acts at 270° . What is the magnitude of the resultant in this case? How does it compare to the resultant in Part 1?
6. As stated earlier, the pulleys introduce a possible source of error because of their possible friction. Given that they are a source of error, why are the pulleys used at all? In other words, what is the function of the pulleys?