Name ______________________ Section ____________ Date ____________

Laboratory 2

Measurement of Density

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and the section entitled "General Laboratory Information" and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. A cylinder has a length of 3.23 cm, a diameter of 1.75 cm, and a mass of 65.3 g. What is the density of the cylinder? Based on its density, of what kind of material might it be made? Show your work.

2. Figure 2.1 shows a vernier caliper scale set to a particular reading. What is the reading of the scale? Reading = ________ cm

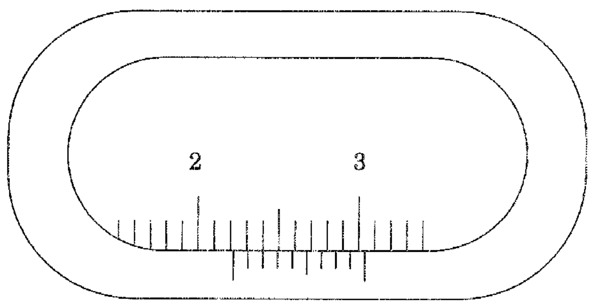

Figure 2.1 Example of a reading of a vernier caliper.

3. The caliper in Figure 2.2 has its jaws closed. If the caliper has a zero error, what is its value, and is it positive or negative? Error = ________ cm

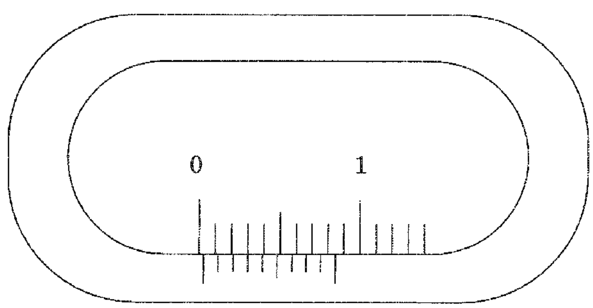

Figure 2.2 Vernier caliper with its jaws closed. Does it have a zero error?

4. A series of four measurements of the mass, length, and diameter of a cylinder are made. The results of these measurements are:

   Mass — 20.6, 20.5, 20.6, and 20.4 g

   Length — 2.68, 2.67, 2.65, and 2.69 cm

   Diameter — 1.07, 1.05, 1.06, and 1.05 cm

Find the mean and standard error for each of the measured quantities and tabulate them below. Keep only *one* significant figure in each standard error and then keep *decimal places* in the mean to coincide with the standard error.

$\overline{M}$ = ________ α_M = ________

$\overline{L}$ = ________ α_L = ________

$\overline{d}$ = ________ α_d = ________

Calculate the density and the standard error of the density using equations 3 and 5. Keep only *one* significant figure in the standard error and then keep *decimal places* in the density to coincide with the standard error.

ρ = ________ α_ρ = ________

5. Assuming α_ρ is a measure of the uncertainty in the density, which is the first uncertain digit in the value for the density given in question 4?

6. Assume that only the first measurement for each quantity given in question 4 was measured. From those single values of mass, length, and diameter, calculate the density. How many significant figures would be justified based on the significant figures of the measured quantities?

Laboratory 2
Measurement of Density

OBJECTIVES

The density of an object is defined as its mass per unit volume. Although the standard SI units for density are kg/m^3, it is also very common to use units of g/cm^3. In this laboratory, measurements on several cylinders of different metals will be made to accomplish the following objectives:

1. Determination of the mass of the cylinders
2. Determination of the lengths and diameters of the cylinders
3. Calculation of the density of the cylinders and comparison with the accepted values of the density of the metals
4. Determination of the uncertainty in the value of the calculated density caused by the uncertainties in the measured mass, length, and diameter

EQUIPMENT LIST

1. Three solid cylinders of different metals (aluminum, brass, and iron). (These cylinders should be cut from a metal rod by hand on a band saw in order to introduce some uncertainty in their lengths. It might also be useful if they were ground nonuniformly along their length in order to introduce uncertainty in their diameters.)
2. Vernier calipers
3. Laboratory balance and calibrated masses

THEORY

The goal of this laboratory is to measure the density of three metal cylinders. In fact the process involves measuring directly the mass, length, and diameter of a cylinder and calculating the density from these directly measured quantities. The most general definition of density is mass per unit volume, which can vary throughout the body if the mass is distributed nonuniformly. If the mass in an object is distributed uniformly throughout the object, the density ρ is defined as the total mass M divided by the total volume V of the object. In equation form this is

$$\rho = \frac{M}{V} \tag{1}$$

For a cylinder the volume is given by

$$V = \frac{\pi d^2 L}{4} \tag{2}$$

where d is the cylinder diameter and L is its length. Using equation 2 in equation 1 gives

$$\rho = \frac{4M}{\pi d^2 L} \tag{3}$$

The quantities M, d, and L will be determined by measuring each of them four times independently and calculating the mean and standard error for each quantity. Using the mean of each measured quantity in equation 3 leads to the best value for the measured density ρ. In addition, the uncertainty in the measurement will be determined.

As discussed in the "General Laboratory Information" section, the standard error of ρ is related to the standard error of M, d, and L by

$$(\alpha_\rho)^2 = \left(\frac{\partial \rho}{\partial M}\right)^2 (\alpha_M)^2 + \left(\frac{\partial \rho}{\partial L}\right)^2 (\alpha_L)^2 + \left(\frac{\partial \rho}{\partial d}\right)^2 (\alpha_d)^2 \tag{4}$$

where $\frac{\partial \rho}{\partial M}$ stands for the partial derivative of ρ with respect to M and similarly for the other quantities. The symbol for the standard error for the measurements of M and for the other quantities is α_M. Those who have studied calculus should be able to use equations 3 and 4 to arrive at the following:

$$\alpha_\rho = \rho \sqrt{\left(\frac{\alpha_M}{M}\right)^2 + \left(\frac{\alpha_L}{L}\right)^2 + 4\left(\frac{\alpha_d}{d}\right)^2} \tag{5}$$

If you have no knowledge of calculus do not be intimidated by its use here. Simply accept without proof that equation 5 is the proper relationship between the uncertainty of the measured quantities M, d, and L and the uncertainty in the quantity ρ calculated from them.

The mass of the cylinders will be determined by a laboratory balance. There are several common versions, but the basic principle is essentially the same for all of them. They balance the weight of an unknown mass m against the weight of a known mass m_k. Although the balance is between two forces (the weight of the masses), the scales can be calibrated in terms of mass, assuming that the force per unit mass is the same for both the known and unknown mass. A common form known as the Harvard Trip balance has a calibrated beam along which a permanent sliding weight can be moved in units of 0.1 g up to 10 g. The unknown mass is placed on a pan at the left and balanced against the sum of the permanent sliding weight and additional known masses placed on a pan at the right. The mass of the unknown is the sum of all the known masses placed on the right pan plus the mass equivalent of the permanent sliding mass on the beam when the scales are balanced. Figure 2.3 shows a picture of a Harvard Trip balance.

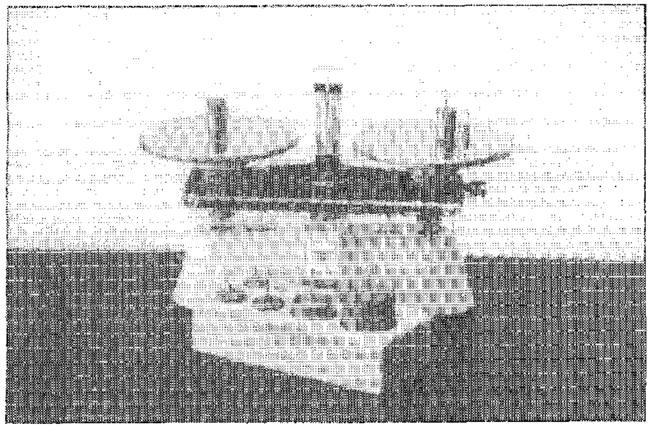

Figure 2.3 Harvard Trip balance.

Before any measurements are made, the balance should be zeroed. Adjustments can be made in the position of a nut on a horizontal screw to ensure that the balance occurs with no mass on either pan when the permanent sliding weight is at the zero position.

The length and diameter of the metal cylinder will be measured with a vernier caliper. Actually, a caliper is any device used to determine thickness, the diameter of an object, or the distance between two surfaces. Often they are in the form of two legs fastened together with a rivet, so they can pivot about the fastened point. The form used in this laboratory consists of a fixed rule containing one jaw and a second jaw with a vernier scale that slides along the fixed-rule scale. Each of the two jaws has two parts pointing in opposite directions. The span between the upper jaws is used to measure the inside diameter between two surfaces. As an example, the upper jaws can be used to measure the inside diameter of a hollow cylinder. The distance between the lower jaws is a measure of the outside diameter of objects over which it is placed.

The caliper has marked on the main scale 1-cm major divisions for which there is both a mark and a number. On the main scale are also marked ten 1-mm divisions between the 1-cm divisions. The 1-mm marks are not labeled with a number. *Vernier* is the name given to any scale that aids in interpolating between marked divisions. This vernier is marked with a scale whose alignment with different marks on the fixed-rule scale allows interpolation between the 1-mm marks on the fixed scale to 0.1-mm accuracy or, in other words, to the nearest 0.01-cm accuracy. A vernier caliper is shown in Figure 2.4.

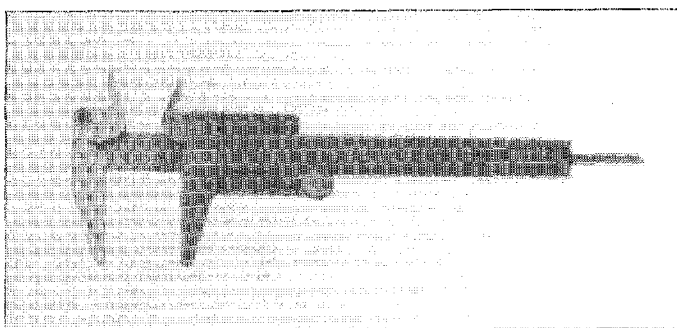

Figure 2.4 Vernier calipers. (Photo courtesy of Sargent-Welch Scientific Co.)

A measurement is made by closing the jaws on some object and noting the position of the zero mark on the vernier and which one of the vernier marks is aligned with some mark on the fixed-rule scale. This is illustrated in Figure 2.5. The position of the zero mark of the vernier scale gives the first two significant figures (2.0 cm in Figure 2.4), and the interpolation between 2.0 cm and 2.1 cm for this case is given by the fact that the sixth mark beyond the vernier zero is best aligned with a mark on the fixed-rule scale. Thus, the reading in this case is 2.06 cm. Study this example carefully to be sure that the instructions on how to read the vernier caliper are clear. If the example is not clear, consult your instructor for additional help in reading these scales.

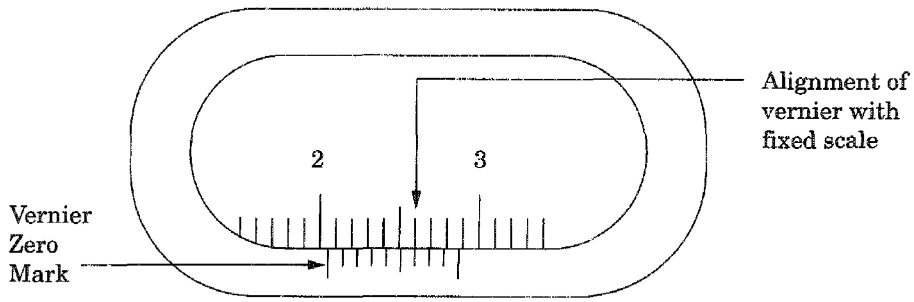

Figure 2.5 Illustration of vernier caliper reading of 2.06 cm.

Before making any measurements it should be determined whether the vernier calipers read zero when the jaws are closed. If the calipers do not read zero when the jaws are closed, they are said to have a zero error. A correction must be made for each measurement made with the calipers. If the vernier zero is to the right of the fixed-scale zero when the jaws are closed, the zero error is negative. Note the mark on the vernier scale that is aligned with the fixed scale, and subtract that number of 0.01-cm units from each measurement. For example, if the third mark to the right of the vernier zero is aligned with the fixed scale when the jaws are closed, then each measurement should have 0.03 cm subtracted from it. On the other hand, if the vernier zero is to the left of the fixed-scale zero, then the zero error is positive. In that case, find which vernier mark is aligned with the fixed scale. Then the number of places to the left of the 10 mark on the vernier scale at which the alignment occurs is the number of 0.01-cm units to be added to the reading. If, for example, the alignment occurs at the 7 mark on the vernier scale, since that is three marks to the left of the 10 mark, then 0.03 cm should be added to the reading.

EXPERIMENTAL PROCEDURE

1. Using the laboratory balance and calibrated masses, determine the mass of each of the three cylinders. Make four independent measurements for each of the cylinders and record the results in the Data Table.
2. Make four separate readings of the zero correction for the vernier calipers. Record the four values in the Data Table. Record the zero correction as positive if the vernier zero is to the right of the fixed-scale zero. Record it as negative if the vernier zero is to the left of the fixed-scale zero.

3. Using the vernier calipers, measure the lengths of the three cylinders. Make four separate trials of the measurement of the length of each cylinder. Measure the length at different places on each cylinder for the four trials in order to sample the variation in length of the cylinders. Record the results in the Data Table.

4. Using the vernier calipers, measure the diameters of the three cylinders. Make four separate trials of the measurement of the diameter of each cylinder. Measure the diameter at four different positions along the length of the cylinders in order to sample the variation in diameter of the cylinders. Record the results in the Data Table.

CALCULATIONS

1. Calculate the mean $\overline{M}$ and the standard error α_M for the four measurements of the mass of each cylinder and record the results in the Calculations Table. Keep only *one* significant figure for all standard errors and then keep the number of *decimal places* in the mean that coincides with the *decimal places* of the standard error.

2. Determine the measured length and diameter for each trial by making the appropriate zero correction to each measurement and then calculating the means $\overline{d}$ and $\overline{L}$ and the standard errors α_d and α_L for each cylinder. Record the results in the Calculations Table. Again keep only *one* significant figure in the standard error and then keep the number of *decimal places* in the mean that coincides with the *decimal places* in the standard error.

3. Using equation 3, calculate the density ρ of each of the cylinders. Use the mean values for the mass, diameter, and length. Calculate the standard error of the density ρ using equation 5. Record the results in the Calculations Table. Again keep only *one significant figure* in the standard error and then keep the number of *decimal places* in the mean that coincides with the *decimal places* in the standard error.

4. For purposes of this laboratory, assume that the density of aluminum is 2.70 g/cm^3, the density of brass is 8.40 g/cm^3, and the density of iron is 7.85 g/cm^3. Calculate the percentage error in your results for the density of each of these metals.

Name ______________________ Section __________ Date __________

Laboratory 2

Measurement of Density

LABORATORY REPORT

Data Table

	Trial 1	Trial 2	Trial 3	Trial 4
Mass–Aluminum				
Mass–Brass				
Mass–Iron				
Zero Reading				
Length–Aluminum				
Length–Brass				
Length–Iron				
Diameter–Aluminum				
Diameter–Brass				
Diameter–Iron				

Calculations Table

Cylinder	$\overline{\overline{M}}$	α_M	$\overline{d}$	α_d	$\overline{L}$	α_L	ρ	α_ρ
Aluminum								
Brass								
Iron								

% error Aluminum = ________ % error Brass = ________ % error Iron = ________

SAMPLE CALCULATIONS

QUESTIONS

1. Consider the uncertainty in the measured value of ρ to be given by α_ρ. Taking the first significant figure of α_ρ as the decimal place that is uncertain, how many significant figures are indicated for each of the measurements of ρ?

2. If the number of significant digits in the density were determined by consideration only of the number of significant digits in the measured quantities, how many significant digits would there be in the measured density?

3. Do the measured values of the density agree with the accepted values of the density of these metals within the standard error α_ρ? Discuss the significance of this question with respect to whether or not there are systematic errors present.

4. For the same percentage error in each of the three quantities–mass, diameter, and length–which would contribute the most to the error in the density? (*Hint*: Consider the form of equation 5.)

5. Calculate the volume (m^3) of each of the three cylinders.