Name ______________________ Section __________ Date __________

Laboratory 19

The Pendulum—Approximate Simple Harmonic Motion

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. What requirement must be satisfied by a force acting on a particle in order for the particle to undergo simple harmonic motion?

2. A particle of mass $M = 1.35$ kg has a force exerted on it such that $F = -0.850x$, where x is the displacement of the particle from equilibrium. The force F is in Newtons, and x is in meters. What is the period T of its motion? Show work.

3. A simple pendulum of length $L = 0.800$ m has a mass $M = 0.250$ kg. What is the tension in the string when it is at an angle of $\theta = 12.5°$? Show work.

4. In question 3, what is the component of the weight of M directed along the arc of the motion of M? Show work.

5. What is the period T of the motion of the pendulum in question 3? Assume that the period is independent of θ.

6. What would be the period T of the pendulum in question 3 if L was unchanged but $M = 0.500$ kg? Assume that the period is independent of θ.

7. Determine the period T of the pendulum in question 3 if everything else stays the same, but $\theta = 45.0°$. Do not assume that the period is independent of θ.

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OBJECTIVES

A bob of mass M suspended on a thin, light string of length L is a good approximation of a simple pendulum. The pendulum is set into motion by displacing the bob until the string makes an angle θ with the vertical and then releasing the bob. The period T of the pendulum is the time for one complete oscillation of the system. Measurements for several such pendulums of varying length L, mass M, and angular amplitude θ will be used to achieve the following objectives:

1. Verification that the period of a pendulum is independent of the mass M of the bob
2. Determination of how the period T of a pendulum depends on the length L of the pendulum
3. Verification that the period T of a pendulum depends very slightly on the angular amplitude of the oscillation for large angles, but that the dependence is negligible for small angular amplitude of oscillation
4. Determination of an experimental value of the acceleration due to gravity g by comparing the measured period of a pendulum with theoretical predictions

EQUIPMENT LIST

1. Pendulum clamp, string, and calibrated hooked masses
2. Laboratory timer
3. Protractor and meter stick

THEORY

A particle of mass M that moves in one dimension is said to exhibit simple harmonic motion if its displacement x from some equilibrium position is described by a single sine or cosine function. This will be the case when the particle is subjected to a force F that is directly proportional to the magnitude of the displacement and directed toward the equilibrium position. In equation form

$$F = -kx \qquad (1)$$

describes a force that produces simple harmonic motion. The period T of the motion is the time for one complete oscillation, and it is determined by the mass M and the constant k. The equation that describes the dependence of the period T on M and k is

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad (2)$$

A pendulum is a system that does not exactly satisfy the above conditions for simple harmonic motion, but it approximates them under certain conditions. An ideal pendulum is a point mass M suspended at one end of a massless string with the other end of the string fixed as shown in Figure 19.1. The motion of the system takes place in a vertical plane when the mass M is released from an initial angle θ. The angular amplitude θ is defined by the angle that the string makes with the vertical.

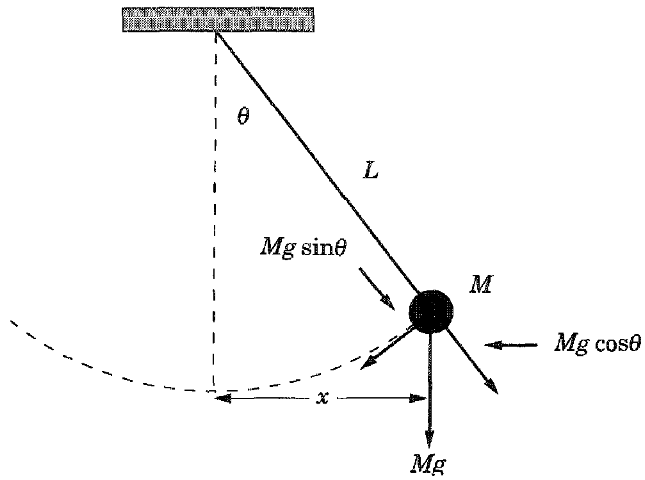

Figure 19.1 Force components acting on the mass bob of a simple pendulum.

The weight of the pendulum acts downward, and it can be resolved into two components. The component $Mg\cos\theta$ is equal in magnitude to the tension N in the string. The other component $Mg\sin\theta$ acts tangent to the arc along which the mass M moves. It is this component that provides the force that drives the system. In equation form, the force F along the direction of motion is

$$F = -Mg\sin\theta \tag{3}$$

For small values of the initial angle θ, the approximation $\sin\theta \approx \tan\theta \approx x/L$ can be used in equation 3, and it then becomes

$$F = -\frac{Mg}{L}x \tag{4}$$

Although equation 4 is an approximation, it is now of the same form as equation 1 with the constant $k = Mg/L$. Using that value of k in equation 2 gives

$$T = 2\pi\sqrt{\frac{M}{Mg/L}} = 2\pi\sqrt{\frac{L}{g}} \tag{5}$$

Equation 5 makes the theoretical prediction that the period T of a simple pendulum is independent of the mass M and the angular amplitude θ and depends only on the length L of the pendulum.

The exact solution to the period of a simple pendulum without making the small-angle approximation leads to an infinite series of terms with each successive term becoming smaller and smaller. Equation 6 gives the first three terms in the series. This is sufficient to determine the very slight dependence that does exist of the period T on the angular amplitude of the motion.

$$T = 2\pi\sqrt{\frac{L}{g}}\,[1 + 1/4\,\sin^2\left(\frac{\theta}{2}\right) + 9/64\,\sin^4\left(\frac{\theta}{2}\right) + \ldots] \tag{6}$$

For an ideal pendulum with no friction, the motion repeats indefinitely with no reduction in the amplitude as time goes on. For a real pendulum, there will always be some friction, and the amplitude of the motion decreases slowly with time. However, for small initial amplitudes, the change in the period as the amplitude decreases is negligible. This fact is the basis for the pendulum clock. Pendulum clocks, in one form or another, have been used for over 300 years. For over 100 years, devices to compensate for small changes in the length of the pendulum caused by temperature variations have been successfully used to build extremely accurate clocks.

EXPERIMENTAL PROCEDURE—LENGTH

1. The dependence of the period on the length of the pendulum will be determined with a constant mass and constant angular amplitude. Place a 0.2000-kg hooked calibrated mass on a string with a loop in one end. Adjust the position at which the other end of the string is clamped in the pendulum clamp until the distance from the point of support to the center of mass of the hooked mass is 1.0000 m. Note carefully that the length L of each pendulum will be from the point of support to the center of mass of the bob. The center of mass of the hooked masses will usually not be in the center because the hooked masses are not solid at the bottom. Estimate how much this tends to raise the position of the center of mass and mark the estimated center of mass on the hooked mass.
2. Displace the pendulum 5.0° from the vertical and release it. Measure the time Δt for 10 complete periods of motion and record that value in Data Table 1. It is best to set the pendulum in motion, and then begin the timer as it reaches the maximum displacement, counting 10 round trips back to that position. Repeat this process two more times, for a total of three trials with this same length. The pendulum should move in a plane as it swings. A tendency for the mass to move in an elliptical path will lead to error.
3. Repeat the procedure of step 2, using the same mass and an angle of 5.0° for pendulum lengths of 0.8000, 0.6000, 0.5000, 0.3000, 0.2000, and 0.1000 m. Do three trials at each length. Remember that the length of the pendulum is from the point of support to the center of mass of the hooked mass.

CALCULATIONS—LENGTH

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t of the three trials for each of the lengths. Record those results in Calculations Table 1.
2. Calculate the period T from $T = \overline{\Delta t} / 10$ and record in Calculations Table 1.
3. According to equation 5, the period T should be proportional to $\sqrt{L}$. For each of the values of the length L calculate the $\sqrt{L}$ and record the results in Calculations Table 1. Perform a linear least squares fit to the data with T as the ordinate and $\sqrt{L}$ as the abscissa. According to equation 5, the slope of this fit should be equal to $2\pi/\sqrt{g}$. Equate the value of the slope determined from the least squares fit to $2\pi/\sqrt{g}$ treating g as unknown. Solve this equation for g and record that value as g_{exp} in Calculations Table 1. Also record the value of the correlation coefficient for the least squares fit.

EXPERIMENTAL PROCEDURE—MASS

1. The dependence of the period T on the mass M of the pendulum will be determined with the length L and amplitude θ held constant. Place a 0.0500-kg hooked mass on the end of the string and adjust the point of support of the string until the pendulum length is 1.0000 m. Displace the pendulum 5.0° and release it. Measure the time Δt for 10 complete periods of the motion and record it in Data Table 2. Repeat this measurement two more times for a total of three trials.
2. Keeping the length constant at $L = 1.0000$ m, repeat the procedure above for pendulum masses of 0.1000, 0.2000, and 0.5000 kg. Because the length of the pendulum is from the point of support to the center of mass, slight adjustments in the string length will be needed to keep the pendulum length constant for the different hooked masses.

CALCULATIONS—MASS

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t of the three trials for each of the masses. Record those results in Calculations Table 2.
2. Calculate the period T from $T = \overline{\Delta t}/10$ and record in Calculations Table 2.

EXPERIMENTAL PROCEDURE—AMPLITUDE

1. The dependence of the period T on the amplitude of the motion will be determined with the length L and mass M held constant. Construct a pendulum 1.0000 m long with a mass of 0.2000 kg. Measure the time Δt for 10 complete periods of the motion with an amplitude of 5.0°. Repeat the measurement two more times for a total of three trials at this amplitude. Record all results in Data Table 3.
2. Repeat the procedure above for amplitudes of 10.0°, 20.0°, 30.0°, and 45.0°. Do three trials for each amplitude and record the results in Data Table 3.

CALCULATIONS—AMPLITUDE

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t of the three trials for each of the amplitudes. Record those results in Calculations Table 3.
2. Calculate the period T from $T = \overline{\Delta t}/10$ and record the results in Calculations Table 3 as T_{exp}.
3. Equation 6 is the theoretical prediction for how the period T should depend on the amplitude. Using the values of $L = 1.000$ m and $M = 0.2000$ kg in equation 6, calculate the value of T predicted for each of the values of the amplitude θ. Record those values in Calculations Table 3 as T_{theo}.
4. For the experimental values of the period T_{exp}, calculate the ratio of the period at the other angles to the period at $\theta = 5.0°$. Call this ratio $T_{exp}(\theta)/T_{exp}(5.0°)$. Record these values in Calculations Table 3.
5. For the theoretical values of the period T_{theo}, calculate the ratio of the period at the other angles to the period at $\theta = 5.0°$. Call this ratio $T_{theo}(\theta)/T_{theo}(5.0°)$. Record these values in Calculations Table 3.

GRAPHS

1. Consider the data for the dependence of the period T on the length L. Graph the period T as the ordinate and $\sqrt{L}$ as the abscissa. Also show on the graph the straight line obtained by the linear least squares fit to the data.

2. Consider the data for the dependence of the period T on the mass M. Graph the period T as the ordinate and the mass M as the abscissa.

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LABORATORY REPORT

Data Table 1

Length (m)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)
1.0000			
0.8000			
0.6000			
0.5000			
0.3000			
0.2000			
0.1000			

Calculations Table 1

L (m)	$\overline{\Delta_t}$ (s)	α_t (s)	T (s)	$\sqrt{L}$ ($\sqrt{m}$)
1.0000				
0.8000				
0.6000				
0.5000				
0.3000				
0.2000				
0.1000				
Slope =		g_{exp} =		r =

Data Table 2

Mass (kg)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)
0.0500			
0.2000			
0.2000			
0.5000			

Calculations Table 2

Mass (kg)	$\overline{\Delta t}$ (s)	α_t (s)	T (s)
0.0500			
0.1000			
0.2000			
0.5000			

Data Table 3

Angle	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)
5.0°			
10.0°			
20.0°			
30.0°			
45.0°			

Calculations Table 3

Angle	$\overline{\Delta t}$ (s)	α_t (s)	T_{exp} (s)	T_{theo} (s)	$\frac{T_{exp}(\theta)}{T_{exp}(5°)}$	$\frac{T_{theo}(\theta)}{T_{theo}(5°)}$
5.0°						
10.0°						
20.0°						
30.0°						
45.0°						

SAMPLE CALCULATIONS

QUESTIONS

1. In general, what is the precision of the measurements of T? Answer this question by considering what percentage is α_t of $\overline{\Delta t}$ for the measurements as a whole.

2. Do your data confirm the expected dependence of the period T on the length L of a pendulum? Consider the correlation coefficient for the least squares fit in your answer.

3. Comment on the accuracy of your experimental value for the acceleration due to gravity g.

4. What does the theory predict for the shape of the graph of period T versus M? Do your data confirm this expectation? Also calculate the mean and standard error of the periods for the four masses and comment on how this relates to mass independence of T.

5. Do your measured values for the period T as a function of the amplitude θ confirm the theoretical predictions? State clearly what is expected and what your data show.

6. The measurements of the period T were done by measuring the time for 10 periods. Why is the time for more than one period measured? If there is an advantage to measuring for 10 periods, why not measure for 1000 periods? Other than the fact that it would take too long, is there a valid reason why measuring for 1000 periods is not a good idea?