PHY200/230 Experiment – Rotational Inertia

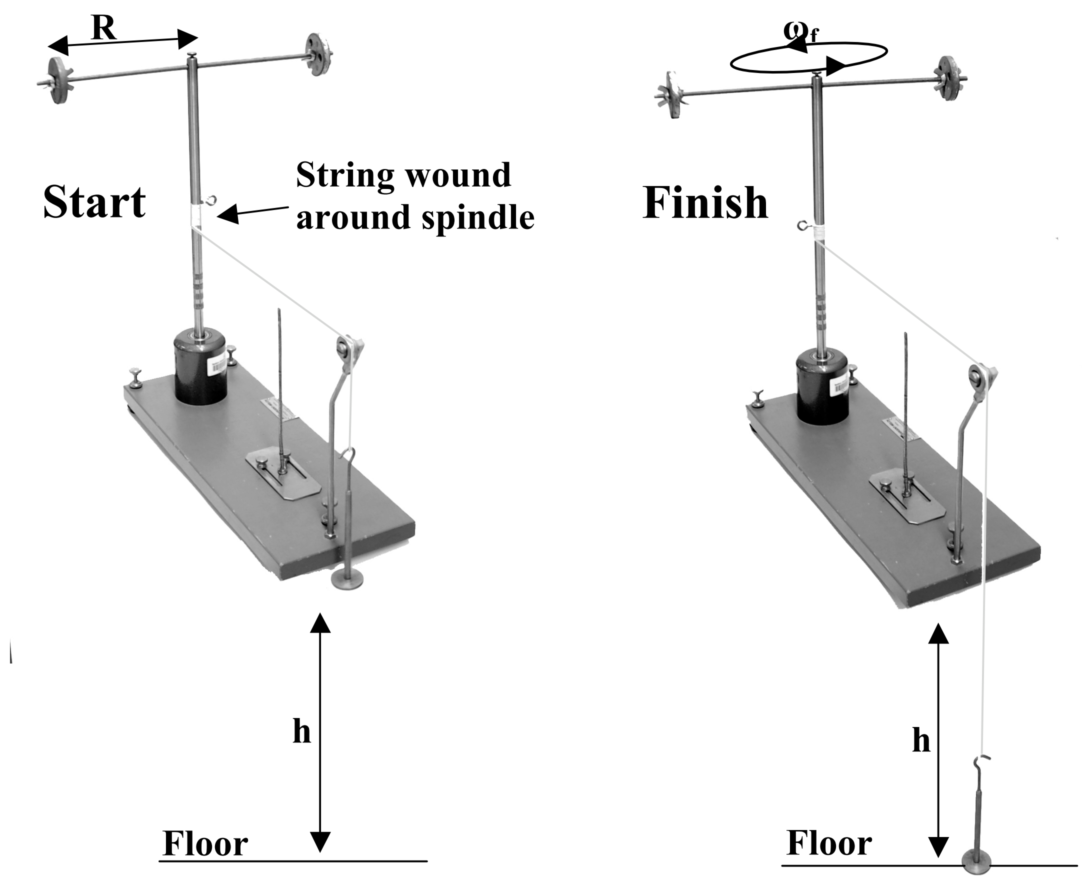
Purpose: To study the relationship between the moment of inertia (I) and rotation arm (R), and to learn to determine moment of inertia experimentally.
Apparatus: Central force/Rotational Inertia apparatus set up for the latter, slotted disk masses, mass hanger, timer, and meter stick.
Discussion: The moment of inertia of an object "I", is directly analogous to the mass of an object as seen in the rotational equivalent to Newton's second law of motion:

$$torque, \tau = I \times \alpha$$

The rotational form of the kinetic energy equation then is

$$KE_{rot} = \frac{1}{2} I \omega^2$$

Consequently, the moment of inertia of a bicycle wheel, rolling ball, or flywheel in a machine is a quantity that is as important as the mass of an object in translational motion. The following derivations will use the energy method although the same results could be obtained using torques and rotational accelerations.



At the beginning of the experiment, the energy is all in the form of gravitational potential energy of the small falling mass, GPE=mgh. At the end of the experiment all of this energy has been transformed into (kinetic energy of the falling mass) + (rotational kinetic energy of the apparatus). Consequently we have:

$$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

This equation will be transformed into one which involves only “I” and other easily measured quantities.

The following definitions are used:

- t = time required for small mass to fall
- h = original height of falling mass
- m =falling mass
- r = radius of central spindle

First, we use the definition of the average velocity of the falling mass:

$$v_{avg} = \frac{h}{t}$$

But, for an object with uniform acceleration, the final velocity is just twice the average velocity. Consequently

$$v_f = \frac{2h}{t}$$

Substituting, the first equation becomes:

$$mgh = \frac{1}{2}m\left(\frac{2h}{t}\right)^2 + \frac{1}{2}I\omega_f^2$$

Next the relationship $v_f = (\omega_f)\,(r)$ is used to eliminate the angular velocity. This is valid because the outer perimeter of the spindle is moving at the same speed as the falling mass (because they are connected by a string that isn’t stretching or going slack). Making the substitution we have:

$$mgh = \frac{1}{2}m\left(\frac{2h}{t}\right)^2 + \frac{1}{2}I\left(\frac{2h}{tr}\right)_f^2$$

Solving for “I”:

$$I = mr^2\left[\left(\frac{gt^2}{2h}\right) - 1\right]$$

This will be used to experimentally determine the moment of inertia of the rotating apparatus.

Data Page: Rotational Inertia Experiment

r = ________________ **Remember: <u>Clear</u> sample Calcs**
$m_{\text{four wing nuts}}$ =_________________ **on separate sheet.**

	First Run	SecondRun	Third Run	Fourth Run	Fifth Run	Empty Run
$m_{fallingmass}$=						
h =						
R = position of masses on arm						
t_{fall}= Trial 1						
Trial 2						
Trial 3						
$t_{fallaverage}$=						
I_{total}=						
R^2 =						

Slope of graph = __________________________ Intercept of Graph =______________
Total rotating mass added, $m_{nuts\&masses}$=_________ Empty moment of inertia=_________
Percent difference =__________________ Percent difference =_____________________

Give suggestions on how this experiment could be more accurate. **(On a separate sheet)**

$I_{nuts\&masses}$ = $I_{total} - I_{empty}$					
$I_{nuts\&masses}$ = $m_{nuts\&masses}R^2$					
Percent difference =					

Experimental Procedure:

1) Measure the radius of the central angle using a high-precision caliper. This is "r".
2) Determine the mass of four wing nuts used to hold the slotted masses in position on the rotating arms.
3) Use the wing nuts to hold two 100 gram masses near the ends of the rotating arm. There should be one 100 gram mass at each end and they should be equidistant from the center. Record this radius distance "R".
4) Connect a 50 gram weight hanger to the end of the string and wind the string up until the hanger is distance "h" from the floor. Record "h".
5) Release the apparatus and allow the weight hanger to fall to the floor. Record the time of fall. Do this at least three times and calculate the average fall time.
6) Calculate the total moment of inertia, I_{tot} of the rotating apparatus.
7) Repeat steps 3 through 6 for four other values of "R". This means that you will gradually move the 100 gram masses inward toward the center of the apparatus. Each time the two 100 gram masses must be equidistant from the central spindle.
8) Plot a graph of I_{total} versus R^2. If any of your data points look bad, go back and check your measurements.
9) Use the linear least squares fit to find the slope and intercept of your graph. Write these values in the space provided.
10) Since moment of inertia is a scalar quantity it can be easily divided. What we have measured is the total moment of inertia, which consists of two parts; the amount of inertia of the empty apparatus, and the moment of inertia of the slotted masses and wing nuts.

$$I_{total} = I_{empty} + I_{masses\&nuts} = I_{empty} + m_{masses\&nuts}R^2$$

According to the above equation the slope of your graph should be equal to the mass of the slotted mass plus wing nuts. Compare. According to the above equation the intercept of your graph should be equal to the moment of inertia of the empty apparatus. Measure the moment of inertia of the empty apparatus and compare.