Alternating Current LR *Circuits*

OBJECTIVES

- ❑ Investigate the phase angle of a generator current relative to the generator voltage.
- ❑ Demonstrate that real inductors consist of both inductance and resistance, and that they can be represented by a pure inductor L in series with a pure resistance r.
- ❑ Determine the value of L and r for an unknown inductor.

EQUIPMENT LIST

- Sine wave generator (variable frequency, 5 V peak to peak amplitude), resistance box
- A 100-mH inductor (resistance $\approx 350\ \Omega$ to serve as an unknown)
- Alternating current voltmeter (digital readout, high frequency capability), compass, protractor

THEORY

Consider the two circuits shown in Figure 36-1 in which a sine wave generator of frequency f is connected separately to resistor R and then to a pure inductance L. The generator is assumed to have a maximum voltage of V and will thus produce a maximum voltage of V across the **resistor** in circuit (a). It will also produce a maximum voltage of V across the inductor in circuit (b). The voltage across the resistor is related to the current by a relationship like that for direct current circuits, which is

$$V_R = IR \qquad \text{(Eq. 1)}$$

If L is the **inductance** (units H) and $\omega = 2\pi f$ is the **angular frequency** of the generator in rad/s, then the following relationship exists between the voltage V_L and the current I

$$V_L = I\omega L \qquad \text{(Eq. 2)}$$

The quantity ωL is called the inductive reactance, and it has units of Ω.

When an alternating current or voltage is measured in the laboratory on a meter, the number read for the current or voltage must be a time-averaged value. Meters are normally calibrated so that they respond to the root-mean-square value of the current or voltage. A **root-mean-square value** of voltage is designated

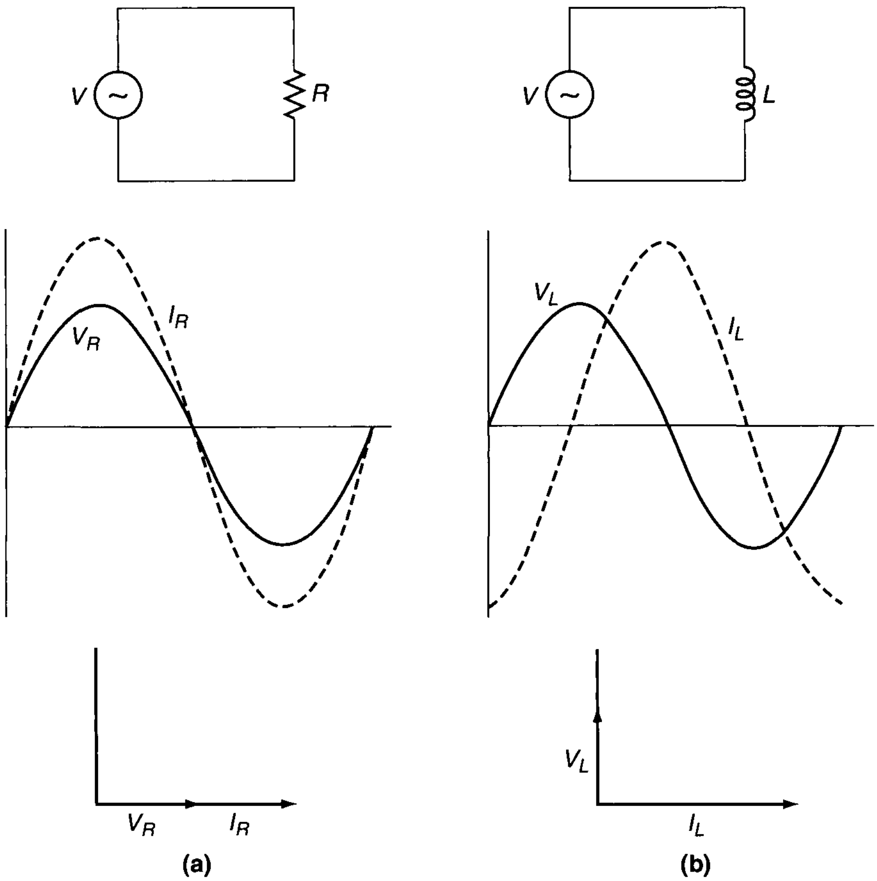

Figure 36-1 Generator and resistor and generator and inductor. Phase relationships between the voltage and current and phasor diagrams of the phase relationships.

as V_{rms}. The relationship between V_{rms} and V the maximum voltage is $V_{rms} = 0.707\,V$. *In this laboratory only voltage will be measured, and all the measurements will be rms values.*

Also shown in Figure 36-1 below each circuit is a graph of the current and voltage across the element for one full period. The graph for the case of the resistor indicates that the resistor current I_R and the resistor voltage V_R are in phase. For the inductor, the graph shows that the inductor current I_L and the inductor voltage V_L are 90° out of phase, with the voltage leading the current by 90°.

Shown at the bottom of Figure 36-1 is a diagram called a **phasor diagram.** Its purpose is also to show the phase relationship. The phasors are vectors drawn with length proportional to the value of the represented quantity, and they are assumed to be rotating counterclockwise with the frequency of the generator. At any time, a projection of one of the rotating vectors on the y axis is the instantaneous value of that quantity. Because the resistor current and voltage are in phase, the phasors are in the same direction. For the inductor, the vector representing the inductor voltage is 90° ahead of the vector representing the current.

Consider now the circuit obtained by placing a pure inductance L having no resistance and a resistor R in series with a sine wave generator of voltage V shown in Figure 36-2.

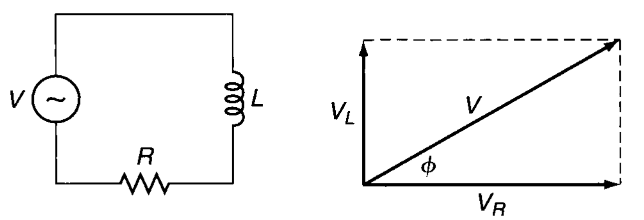

Figure 36-2 Series circuit of resistor and inductor and associated phasor diagram.

For this circuit, the current I is the same at every instant of time in all three circuit elements. Also given in Figure 36-2 is a phasor diagram in which only the voltages are shown. The phasor representing the current (which is not shown) would be in the direction of the phasor labeled V_R because the current and the resistor voltage are in phase. Note that the inductor voltage V_L is 90° ahead of the resistor voltage V_R, and the generator voltage is angle ϕ ahead of V_R. This phasor diagram shows that the generator voltage V is the vector sum of V_R and V_L. In equation form the phasor diagram states

$$V = \sqrt{V_L^2 + V_R^2} \qquad \text{(Eq. 3)}$$

The phasor diagram shows that the phase angle ϕ is related to the voltages V_L and V_R, and thus to the resistance R and ωL through Equations 1 and 2. The relationship is given by

$$\tan\phi = \frac{V_L}{V_R} = \frac{\omega L}{R} \qquad \text{(Eq. 4)}$$

Note that Equation 4 is strictly valid only for a pure inductor that has no resistance. Real inductors have both an inductance L and an internal resistance r, and can be represented by a pure inductance L in series with a pure resistance r. In Figure 36-3 a real inductor is shown in series with a resistor R and a generator of voltage V. The voltage between points A and B is the generator voltage V, and the voltage between A and C is the resistor voltage V_R. Between the points B and C is the combined voltage across the inductance L and the internal resistance r. This voltage will be referred to as V_{ind}. There is some voltage V_L across L, and some voltage V_r across r. However, there can be no direct measurement of V_L or V_r. The only quantity that can be measured is V_{ind}, which is the vector sum of V_L and V_r. A phasor diagram for the circuit is also shown in Figure 36-3.

Applying the law of cosines to the triangle formed by V, V_R, and V_{ind} leads to

$$\cos\phi = \frac{V^2 + V_R^2 - V_{\text{ind}}^2}{2VV_R} \qquad \text{(Eq. 5)}$$

The phasor diagram in Figure 36-3 shows that voltages V_L and V_r can be determined from V, V_R, and ϕ by

$$V_L = V\sin\phi \quad \text{and} \quad V_r = V\cos\phi - V_R \qquad \text{(Eq. 6)}$$

The current I is the same in all the elements of the circuit, and it can be related to the voltage across each element by the following equations:

$$V_L = I\omega L \quad V_R = IR \quad V_r = Ir \qquad \text{(Eq. 7)}$$

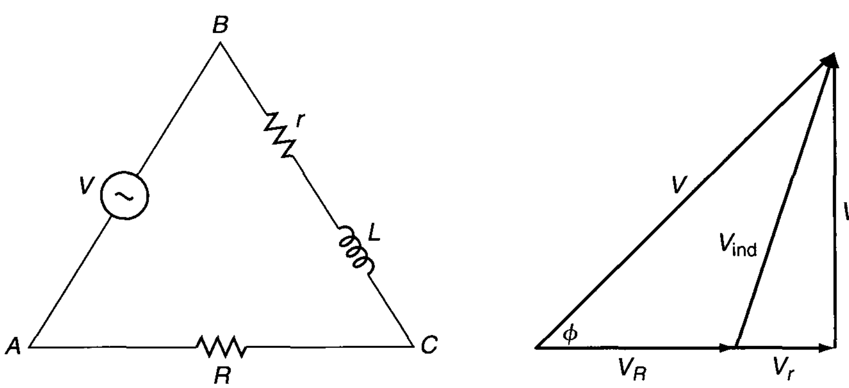

Figure 36-3 Series circuit of inductor with inductance L and internal resistance r, a resistor R, and a generator of voltage V. Also shown is the phasor diagram of the voltages.

With ϕ, V_L, and V_r determined from Equations 5 and 6, Equations 7 can be used to solve for ωL and r by eliminating I to get

$$\omega L = R\frac{V_L}{V_R} \qquad r = R\frac{V_r}{V_R} \qquad \text{(Eq. 8)}$$

EXPERIMENTAL PROCEDURE

1. Connect the inductor in series with the sine wave generator and a resistance box to form a circuit like that of Figure 36-3. Set the generator to maximum voltage and a frequency of 800 Hz. Set the resistance box to a value of 400 Ω and record that value as R and the frequency f in the Data Table.
2. Using the alternating current voltage scale on the voltmeter, measure the generator voltage V, the inductor voltage V_{ind}, and the resistor voltage V_R. Record these values in the Data Table.
3. Repeat Steps 1 and 2 for R of 600, 800, and 1000. Even though the voltage setting is left at the maximum setting, the generator output might change slightly in response to the changes in R. Therefore, be sure to measure all three voltages for each value of R.
4. Make careful note of the particular inductor used and the values of L and r determined. You may need to identify it and use it again in other laboratory exercises.

CALCULATIONS

1. From the known value of the frequency f, calculate and record in the Calculations Table the value of the angular frequency ω ($\omega = 2\pi f$).
2. Use the appropriate equations to calculate $\cos\phi$, ϕ, V_L, V_r, ωL, r, and L for each of the four cases. Record all values in the Calculations Table.
3. Calculate the mean and standard errors for the four values of r and the four values of L and record them in the Calculations Table as $\bar{r}$, $\bar{L}$, α_r, and α_L.

GRAPHS

1. Construct to scale a phasor diagram like the one shown in Figure 36-4 for each of the four cases. Use one sheet of graph paper and make four separate diagrams on the one sheet of paper. Choose a scale (for example, 1.00 V/cm) so that the diagrams are as large as possible, but that each one fits on one-fourth of the sheet of paper. First construct a vector along the x axis with a length scaled to the magnitude of V_R, as shown in Figure 36-4. Use a compass to construct an arc from the end of V_R with a

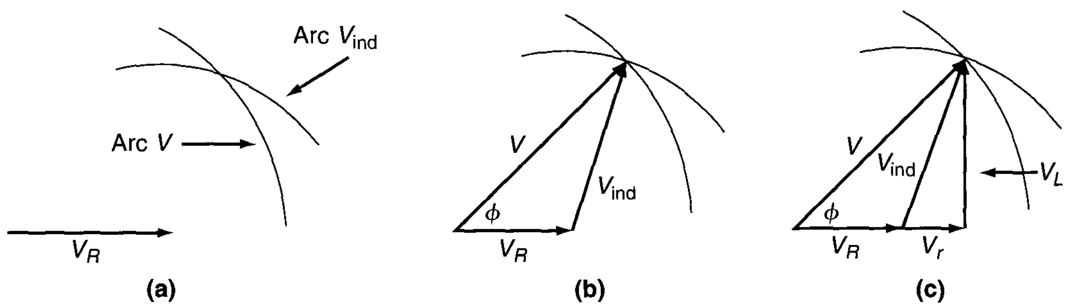

Figure 36-4 Phasor diagram construction.

radius the length of the scaled value of V_{ind}. Finally, construct an arc from the beginning of V_R with a radius the length of the scaled value of V. The intersection of the two arcs is the intersection of V_{ind} and V, and those two vectors can then be drawn in their proper direction as shown in part (b) of the figure. Finally, V_L and V_r can be constructed as shown in part (c) of the figure by dropping a perpendicular from the intersection of the arcs to the x axis and extending a vector from the end of V_R.

Name ______ Section ______ Date ______

LABORATORY 36 *Alternating Current* LR *Circuits*

PRE-LABORATORY ASSIGNMENT

1. For a resistor in a series alternating current circuit, the phase relationship between the current in the resistor and the voltage across the resistor is (a) the current leads the voltage by 90° (b) the voltage leads the current by 90° (c) the current is in phase with the voltage (d) the current is at some phase angle ϕ relative to the voltage (ϕ is dependent on the circuit parameters).

2. For an inductor in a series alternating current circuit, the phase relationship between the current in the inductor and the voltage across the inductor is (a) the current leads the voltage by 90° (b) the voltage leads the current by 90° (c) the current is in phase with the voltage (d) the current is at some phase angle ϕ relative to the voltage (ϕ is dependent on the circuit parameters).

3. For a generator in a series alternating current circuit, the phase relationship between the generator voltage and the current in the generator is (a) the current leads the voltage by 90° (b) the voltage leads the current by 90° (c) the current is in phase with the voltage (d) the current is at some phase angle ϕ relative to the voltage (ϕ is dependent on the circuit parameters).

4. If a generator has a maximum voltage of 5.00 V, what is the root-mean-square voltage of the generator? Show your work.

   V_{rms} = ________ V

5. A 2.50 mH inductor has an rms voltage of 15.0 V across it at a frequency $f = 200$ Hz. What is the rms current in the inductor? Show your work.

   I_{rms} = ________ A

6. A pure inductor L and a pure resistor R are in series with a generator of voltage V. The voltage across the inductor is $V_L = 10.0\,\text{V}$. The voltage across the resistor is 15.0 V. What is the voltage V of the generator? Show your work.

   $V =$ ____________ V

7. A 500 Ω resistor and a real inductor with a pure inductance of L and an internal resistance of r are in series with a generator with a voltage of $V = 10.0\,\text{V}$ and an angular frequency of $\omega = 1000\,\text{rad/s}$. The voltage across the real inductor is measured to be 4.73 V, and the voltage across the 500 Ω resistor is measured to be 6.57 V. What is the value of L and r? (Hint—This is the measurement to be performed in this laboratory exercise. Use the appropriate equation to find ϕ, then the appropriate equations to find V_L and V_r, and then the appropriate equations to find ωL and r. Finally, find L from the known value of ω.) Show your work.

Name _______________ Section _______ Date _______

Lab Partners _______________

LABORATORY 36 *Alternating Current* LR *Circuits*

LABORATORY REPORT

Data Table

$f=$		Hz		
R (Ω)				
V (V)				
V_{ind} (V)				
V_R (V)				

Calculations Table

$\omega = 2\pi f=$		rad/s		
$\cos\phi$				
ϕ (degrees)				
V_L (V)				
V_r (V)				
ωL (Ω)				
r (Ω)				
L (H)				

$\bar{r}=$ Ω	$\alpha_r=$ Ω	$\bar{L}=$ H	$\alpha_L=$ H

SAMPLE CALCULATIONS

1. $\omega = 2\pi f =$
2. $\cos\phi = \dfrac{V^2 + V_R^2 - V_{ind}^2}{2VV_R} =$
3. $\phi = \cos^{-1}(\cos\phi) =$
4. $V_L = V \sin\phi =$
5. $V_r = V\cos\phi - V_R =$
6. $\omega L = (R)(V_L)/V_R =$
7. $r = (R)(V_r)/V_R =$
8. $L = (\omega L)/(\omega) =$

QUESTIONS

1. Comment on the precision of your measurement of L and r. State the evidence for your comments.

2. Examine the phasor diagrams that you have constructed. Using a protractor, measure the angle ϕ of the constructed triangle of V, V_R, and V_{ind}. Compare it with the calculated value of ϕ for each of the phasor diagrams. Calculate the percentage error in the value of ϕ from the diagram compared to the calculated value.

3. If your inductor was used in a series circuit with a resistance of $R = 10{,}000\,\Omega$ and a generator of $\omega = 100{,}000$ rad/s, what would be the phase angle ϕ? (Hint—The resistance of the inductor would be negligible.)

4. Consider the circuit that you measured with $R = 600\,\Omega$. Calculate the value of the current from each of the three Equations 7 and compare their agreement.