Kirchhoff's Rules

OBJECTIVES

❑ Investigate what type of circuit to which Kirchhoff's rules must be applied.

❑ Apply Kirchhoff's rules to several circuits, solve for the currents in the circuit, and compare the theoretical values predicted by Kirchhoff's rules to measured values.

EQUIPMENT LIST

- Direct current voltmeter (digital, 0–20 V), direct current ammeter (digital, 0–1000 mA)
- Two sources of emf (direct current power supplies, up to 12 V)
- Three or four resistors or resistor boxes (range 500–1000 Ω)
- Digital ohmmeter (one for the class), connecting wires

THEORY

Consider the circuit in Figure 34-1. The circuit is labeled with all of the currents. The 2 Ω resistor, 8 Ω resistor, and 12 V power supply have current I_1, the 6 Ω resistor has current I_2, and the 3 Ω resistor has current I_3. This circuit is called a **single-loop circuit** because it can be reduced to a single resistor in series with the power supply. The 6 Ω resistor and the 3 Ω resistor are in parallel with an equivalent resistance of 2 Ω. That equivalent 2 Ω resistance is in series with the 12 V power supply and the other two resistors, reducing the circuit to a single-loop circuit. The total resistance across the 12 V power supply is 12 Ω and its current is therefore $I_1 = 1$ A. Applying Ohm's law to the remaining part of the circuit gives $I_2 = 1/3$ A and $I_3 = 2/3$ A.

Consider now the circuit of Figure 34-2. This laboratory is concerned with the fundamental difference between circuits of the type depicted in Figure 34-1 and circuits of the type depicted in Figure 34-2. The circuit in Figure 34-2 cannot be reduced to a single-loop circuit, but instead is called a **multi-loop circuit.** Before analyzing this circuit, first we will define some terms. A point at which at least three possible current paths intersect is defined as a **junction.** For example, points A and B in Figure 34-2 are junctions. A **closed loop** is any path that starts at some point in a circuit and passes through elements of the circuit (in this case resistors and power supplies), and then arrives back at the same point without passing through any circuit element more than once. By this definition there are three loops in the circuit of Figure 34-2: (1) starting at B, going through the 10 V power supply to A, and then down through the 20 V power supply back to B, (2) starting at B, up through the 20 V power supply, and then around the outside through the 10 Ω resistor and back to B, (3) completely around the outside part of the circuit. One can traverse a loop in either of two directions, but regardless of which direction is chosen, the resulting equations are equivalent.
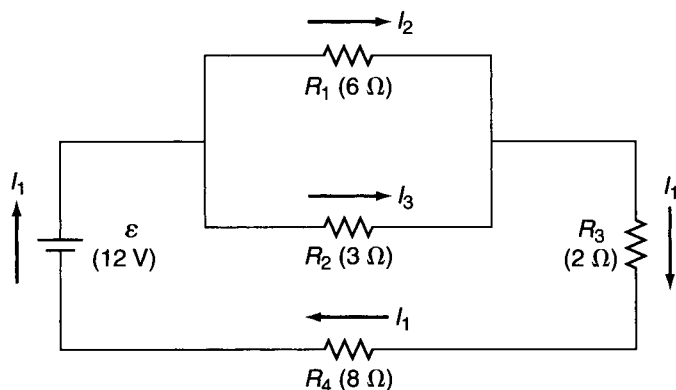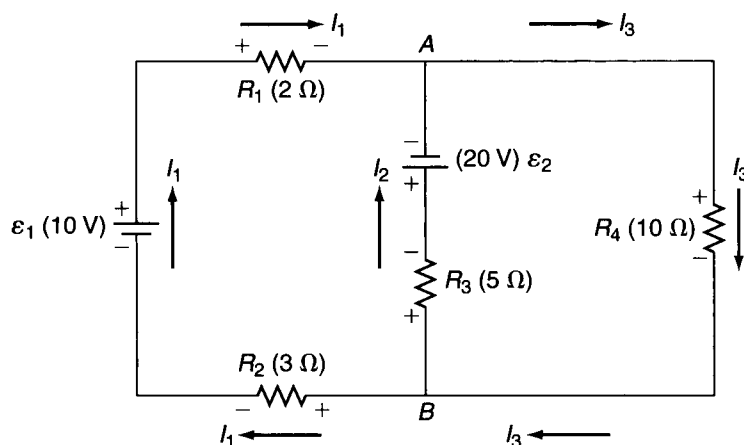
Figure 34-1 Single-loop circuit.

Figure 34-2 Multi-loop circuit.

The solution for the currents in a multi-loop circuit uses two rules developed by Gustav Robert Kirchhoff. The first of these rules is called **Kirchhoff's current rule** (KCR). It can be stated in the following way:

KCR—The sum of currents into a junction = the sum of currents out of the junction.

This rule actually amounts to a statement of conservation of charge. In effect it states that charge does not accumulate at any point in the circuit. The second rule is called **Kirchhoff's voltage rule** (KVR). It can be stated as:

KVR—The algebraic sum of the voltage changes around any closed loop is zero.

This rule is essentially a statement of the conservation of energy, which recognizes that the energy provided by the power supplies is absorbed by the resistors.

In a multi-loop circuit the values of the resistors and the power supplies are known. It is necessary to determine how many independent currents are in the circuit, to label them, and then to assign a direction to each current. Application of Kirchhoff's rules to the circuits, treating the assigned currents as unknowns, will produce as many independent equations as there are unknown currents. Solving those equations will determine the values of the currents.

In the application of KVR to a circuit, take care to assign the proper sign to a voltage change across a particular element. The value of the voltage change across an emf ε can be either $+\varepsilon$ or $-\varepsilon$ depending upon which direction it is traversed in the loop. If the emf is traversed from the $(-)$ terminal to the $(+)$ terminal, the change in voltage is $+\varepsilon$. However, when going from the $(+)$ terminal to the $(-)$ terminal, the change in

voltage is $-\varepsilon$. In the laboratory we will measure the terminal voltage of the sources of emf. We will assume that those values approximate the emf.

When a resistor R with an assumed current I is traversed in the loop in the same direction as the current, the voltage change is $-IR$. If the resistor is traversed in the direction opposite that of the current, the voltage change is $+IR$. The sign of the voltage change across an emf is not affected by the direction of the current in the emf. The sign of the voltage change across a resistor is completely determined by the current direction.

Consider the application of Kirchhoff's rules to the multi-loop circuit of Figure 34-2. At the junction A currents I_1 and I_2 go into the junction, current I_3 goes out of the junction, and KCR states

$$I_1 + I_2 = I_3 \tag{Eq. 1}$$

It might appear that applying KCR to the junction B would produce an additional useful equation, but in fact it would result in an equation that is identical to Equation 1.

Applying KVR to the loop that starts at B, goes through the 10 V power supply to A, and then down through the 20 V power supply back to B, gives the following equation with values of the resistances included.

$$-R_2 I_1 + \varepsilon_1 - R_1 I_1 + \varepsilon_2 + R_3 I_2 = 0 \quad \text{or} \quad -3 I_1 + 10 - 2 I_1 + 20 + 5 I_2 = 0 \tag{Eq. 2}$$

The signs used in Equation 2 and the circuit diagrams are consistent with the description given above for determining the signs of voltage changes. Applying KVR to the loop that starts at B and goes clockwise around the right side of the circuit gives

$$-R_3 I_2 - \varepsilon_2 - R_4 I_3 = 0 \quad \text{or} \quad -5 I_2 - 20 - 10 I_3 = 0 \tag{Eq. 3}$$

Equations 1, 2, and 3 are the three needed equations for the three unknowns I_1, I_2, and I_3. The solution of these equations gives values for the currents of $I_1 = 2.800$ A, $I_2 = -3.200$ A, and $I_3 = -0.400$ A. The currents I_2 and I_3 are negative. This indicates that the original assumption of direction for these two currents was incorrect. The interpretation of the solution is that there is a current of 2.800 A in the direction indicated in the figure for I_1, a current of 3.200 A in a direction opposite to that indicated in Figure 34-2 for I_2, and a current of 0.400 A in a direction opposite to that indicated for I_3. This is a general feature of solutions using Kirchhoff's rules. Even if the original assumption of the direction of a current is wrong, the solution of the equations leads to the correct understanding of the proper direction by virtue of the sign of the current.

EXPERIMENTAL PROCEDURE

1. If using resistance boxes, choose $R_1 = 500\,\Omega$, $R_2 = 750\,\Omega$, and $R_3 = 1000\,\Omega$, but if using standard resistors, choose values as close as possible to the values listed and use the ohmmeter to measure the value of the resistors. Record those values in Data Table 1.

2. Using two power supplies and the resistors R_1, R_2, and R_3, construct a circuit like that shown in Figure 34-3 with $\varepsilon_1 = 10.0$ V and $\varepsilon_2 = 5.00$ V.

3. Measure the currents I_1, I_2, and I_3. Assuming that only one ammeter is available, the currents will have to be measured one at a time by placing the ammeter in the positions shown in the circuit diagram as a circle. Note that placing the ammeter in the circuit with the polarity shown in the circuit diagram will give positive readings when the current is in the direction assumed. If a modern digital ammeter is used for the measurements, the ammeter will give a positive reading if the current is in the direction assumed, and will give a negative reading if the current is in the opposite direction. If an ammeter is used that properly deflects in only one direction, the meter could be damaged if the current is in the opposite direction from that assumed. In this case, bring the voltage of the power supplies up slowly to see that the meter deflects in the proper direction.
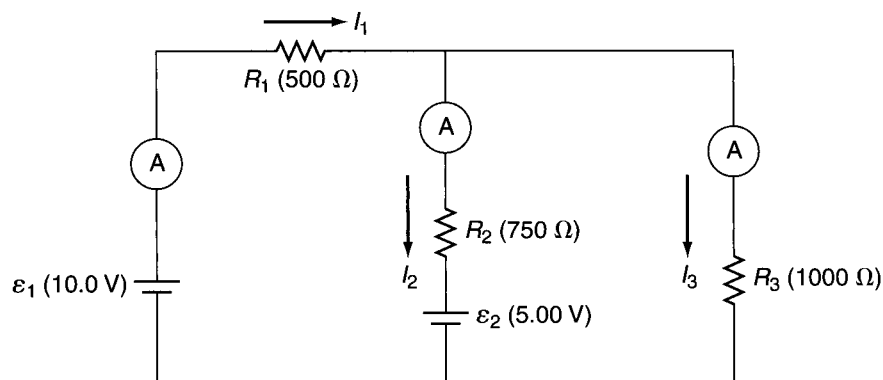
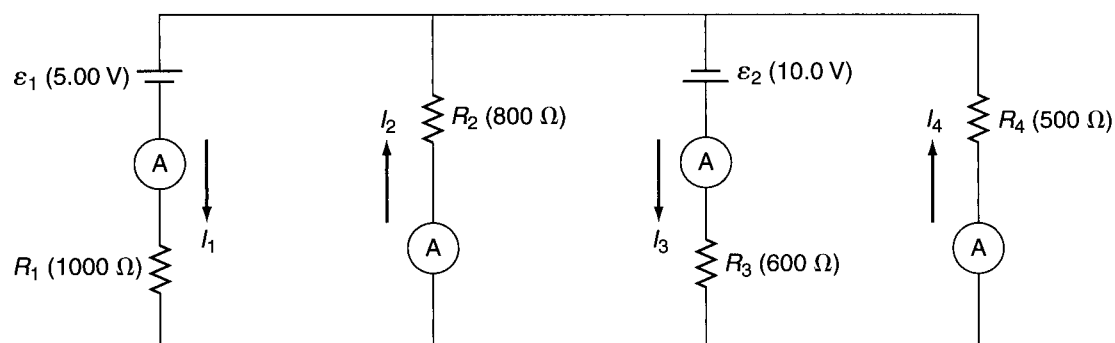**Figure 34-3** Experimental multi-loop circuit with three unknown currents.

Figure 34-4 Experimental multi-loop circuit with four unknown currents.

4. Measure the emfs ε_1 and ε_2 with a voltmeter and record those values in Data Table 1. The terminal voltages of the power supplies are assumed to approximate the emfs.

5. Construct the circuit of Figure 34-4, which also has two power supplies but has four resistors. Choose values of $\varepsilon_1 = 5\,V$, $\varepsilon_2 = 10\,V$, $R_1 = 1000\,\Omega$, $R_2 = 800\,\Omega$, $R_3 = 600\,\Omega$, and $R_4 = 500\,\Omega$, or values as close to those as possible. Determine and record in Data Table 2 the values of the resistors, the four currents, and the emf of the power supplies.

CALCULATIONS

1. Apply Kirchhoff's rules to the circuit of Figure 34-3 for the actual values used in the circuit. Three equations in the three currents I_1, I_2, and I_3 will result. One equation will be a KCR equation, and two will be KVR equations. Record those three equations in the appropriate place in Calculations Table 1.

2. Solve the three equations for the values of I_1, I_2, and I_3 and record the values in Calculations Table 1.

3. Calculate the percentage error of the experimental values of the current compared to the theoretical values for each of the currents and record in Calculations Table 1.

4. Apply Kirchhoff's rules to the circuit of Figure 34-4 with the actual values that were used in your circuit. Four equations in the four currents I_1, I_2, I_3, and I_4 will result. One equation will be a KCR equation, and three will be KVR equations. Record the four equations in the appropriate place in Calculations Table 2.

5. Solve the four equations that you have written for the values of I_1, I_2, I_3, and I_4. Record those values in Calculations Table 2.

6. Calculate the percentage error of the experimental values of the current compared to the theoretical values for each of the currents and record in Calculations Table 2.

LABORATORY 34 *Kirchhoff's Rules*

PRE-LABORATORY ASSIGNMENT

1. Consider the circuit in Figure 34-5. Choose any of the following statements about the circuit that are true. More than one may be correct. (a) It is a single-loop circuit. (b) It is a multi-loop circuit. (c) Assuming R_1 and R_2 were known, the currents could be determined, but only if Kirchhoff's rules were used. (d) Assuming R_1 and R_2 were known, the currents could be determined without the use of Kirchhoff's rules.
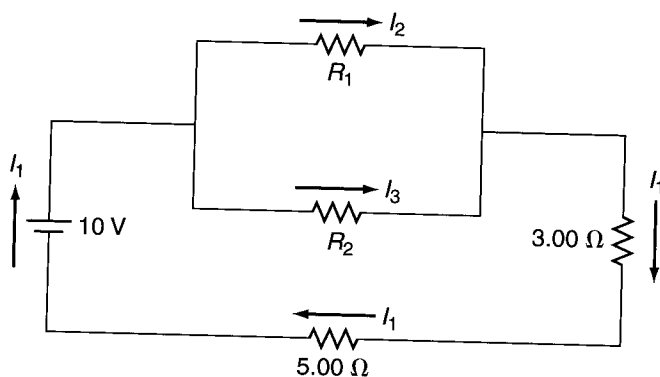

Figure 34-5 Circuit for Questions 1 to 4.

2. In the circuit of Figure 34-5, if $I_1 = 2.00$ A and $I_2 = 0.75$ A, what is the value of I_3?

For questions 3 and 4, assume that the value of R_1 and R_2 in Figure 34-5 are both $4.00\,\Omega$.

3. What is the equivalent resistance of the circuit?

4. What is the current in the 5.00 Ω resistor?

5. Consider the circuit of Figure 34-6. Apply Kirchhoff's rules to the circuit and write three equations in terms of known circuit elements and the unknown currents shown in the figure.
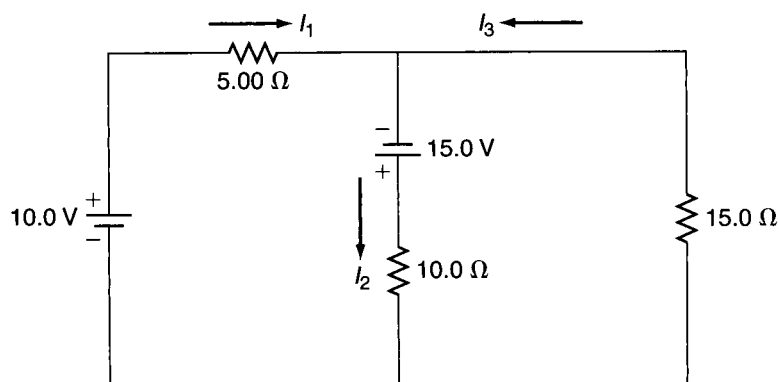

Figure 34-6 Multi-loop circuit.

6. Solve the three equations that you wrote in Question 5 for the values of the currents.

7. If any of the current values obtained in the solution to Question 6 are negative, explain the significance of a negative value for a current.

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LABORATORY REPORT

Data Table 1

Power Supply Voltages	
$\varepsilon_1 =$ _____V	
$\varepsilon_2 =$ _____V	
Resistor Values (Ω)	Experimental Current (mA)
$R_1 =$	$I_1 =$
$R_2 =$	$I_2 =$
$R_3 =$	$I_3 =$

Calculations Table 1

Kirchhoff's rules for the circuit	
(1) KCR—	
(2) KVR1—	
(3) KVR2—	
Theoretical Current (mA)	% Error Experimental to Theoretical Current
$I_1 =$	
$I_2 =$	
$I_3 =$	

Data Table 2

Power Supply Voltages	
$\varepsilon_1 =$ _____V	
$\varepsilon_2 =$ _____V	
Resistor Values (Ω)	Experimental Current (mA)
$R_1 =$	$I_1 =$
$R_2 =$	$I_2 =$
$R_3 =$	$I_3 =$
$R_4 =$	$I_4 =$

Calculations Table 2

Kirchhoff's rules for the circuit	
(1) KCR—	
(2) KVR1—	
(3) KVR2—	
(4) KVR3—	
Theoretical Current (mA)	% Error Experimental to Theoretical Current
$I_1 =$	
$I_2 =$	
$I_3 =$	
$I_4 =$	

SAMPLE CALCULATIONS

1. KCR, KVR1, and KVR2 provide three equations to be solved for I_1, I_2, and I_3 for first circuit.

2. KCR, KVR1, KVR2, and KVR3 provide four equations to be solved for I_1, I_2, I_3, and I_4 for second circuit.

QUESTIONS

1. In Figure 34-3, state the equation that relates the currents I_1, I_2, and I_3. Calculate the percentage difference between the experimental values of the two sides of the equation.

2. In Figure 34-4, state the equation that relates the currents I_1, I_2, I_3, and I_4. Calculate the percentage difference between the experimental values of the two sides of the equation.

3. Are the experimental values of the currents for the entire laboratory generally larger or smaller than the theoretical values expected for the currents?

4. An ideal ammeter has zero resistance. Real ammeters have small but finite resistance. Would ammeter resistance cause an error in the proper direction to account for the direction of your error indicated in Question 3? State your reasoning.

5. The connecting wires in the experiment are assumed to have no resistance, but in fact have a finite resistance. Would this error be in the proper direction to account for the direction of the error stated in your answer to Question 3? State your reasoning.