# *Simple Harmonic Motion— Mass on a Spring*

## OBJECTIVES

- ❑ Directly determine the spring constant $k$ of a spring by measuring the elongation versus applied force.
- ❑ Determine the spring constant $k$ from measurements of the period $T$ of oscillation for different values of mass.
- ❑ Investigate the dependence of the period $T$ of oscillation of a mass on a spring on the value of the mass and on the amplitude of the motion.

## EQUIPMENT LIST

- Spring, masking tape, laboratory timer, meter stick, table clamps, and rods
- Right-angle clamps, laboratory balance, and calibrated hooked masses

## THEORY

A mass that experiences a restoring force proportional to its displacement from an equilibrium position is said to obey **Hooke's law.** In equation form this relationship can be expressed as

$$F = -ky \qquad \text{(Eq. 1)}$$

where $k$ is a constant with dimensions of N/m. The negative sign indicates that the force is in the opposite direction of the displacement. If a spring exerts the force, the constant $k$ is the **spring constant.**

A force described by Equation 1 will produce an oscillatory motion called **simple harmonic motion** because it can be described by a single sine or cosine function of time. A mass displaced from its equilibrium position by some value $A$, and then released, will oscillate about the equilibrium position. Its **displacement** $y$ from the equilibrium position will range between $y = A$ and $y = -A$ with $A$ called the **amplitude** of the motion. For the initial conditions described above, the displacement $y$ as a function of time $t$ is given by

$$y = A\cos(\omega t + \phi) \qquad \text{(Eq. 2)}$$

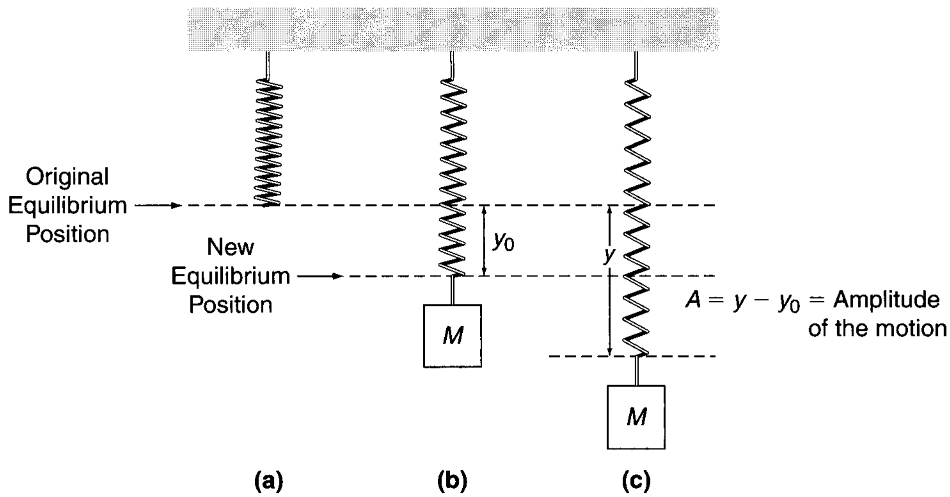


**Figure 20-1** New equilibrium position with mass $M$ placed on a spring.

with angular frequency $\omega$ related to the frequency $f$ and the period $T$ by

$$\omega = \sqrt{\frac{k}{M}} \qquad \omega = 2\pi f \qquad T = 1/f \qquad T = 2\pi\sqrt{\frac{M}{k}} \qquad \text{(Eq. 3)}$$

A mass $M$ placed on the end of a spring hangs vertically as shown in Figure 20-1. The original equilibrium position of the lower end of the spring is shown in Figure 20-1(a). The position of the lower end of the spring when the mass is applied, shown in Figure 20-1(b), can be considered as the new equilibrium position. In Figure 20-1(c) the mass is pulled down to a displacement $A$ from this new equilibrium position. When released, the mass will oscillate with amplitude $A$ and period $T$ given above.

Equation 3 for the period is strictly true only if the spring is massless. For real springs with finite mass, a fraction of the spring mass $m_s$ must be included along with the mass $M$. If $C$ stands for the fraction of the spring mass to be included, the period is

$$T = 2\pi\sqrt{\frac{M + Cm_s}{k}} \qquad \text{(Eq. 4)}$$

You will be challenged to discover what fraction $C$ of the spring mass should be included from your analysis of the data that you will take in the laboratory.

## EXPERIMENTAL PROCEDURE

### Spring Constant

1. Attach the table clamp to the edge of the laboratory table and screw a threaded rod into the clamp vertically as shown in Figure 20-2. Place a right-angle clamp on the vertical rod and extend a horizontal rod from the right-angle clamp. Hang the spring on the horizontal rod and attach it to the horizontal rod with a piece of tape. Screw a threaded vertical rod into a support stand, which rests on the floor. Place a right-angle clamp on the vertical rod and place a meter stick in the clamp so that the meter stick stands vertically. Adjust the height of the clamp on the vertical rod until the zero mark of the meter stick is aligned with the bottom of the hanging spring as shown in Figure 20-2.
2. Place a hooked mass $M$ of 0.050 kg on the end of the spring. Slowly lower the mass $M$ until it hangs at rest in equilibrium when released. Carefully read the position of *the lower end of the spring* on the meter

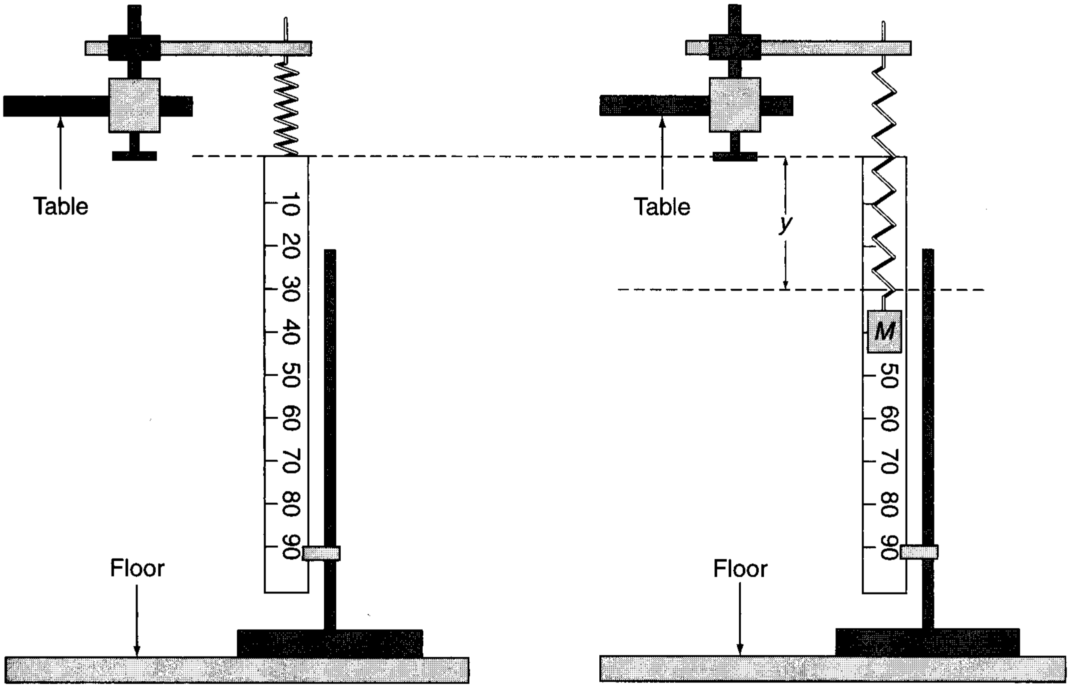


**Figure 20-2** Arrangement to measure displacement of spring caused by mass $M$.

stick scale. Record the value of the mass $M$ and the value of the displacement $x$ in Data and Calculations Table 1.

3. Repeat Step 2, placing in succession 0.100, 0.200, 0.300, 0.400, and 0.500 kg on the spring and measuring the displacement $y$ of the spring. Record all values of $M$ and $y$ in Data and Calculations Table 1.

## Amplitude Variation

1. We will investigate dependence of the period $T$ on the amplitude $A$ for a fixed mass of 0.500 kg. Place the mass on the end of the spring and slowly lower the mass until it hangs at rest when released. Record this position of the lower end of the spring as $y_o$.
2. Displace the mass downward to $y = y_o + 0.0200\,\text{m}$ as shown in Figure 20-1, which will produce $A = 0.0200$ m. Release the mass, and let it oscillate. Measure the time for 10 complete periods and record it in Data Table 2 as $\Delta t$. Repeat the procedure two more times for a total of three trials at this amplitude.
3. Repeat Step 2 above for $A$ of 0.0400, 0.0600, 0.0800, 0.1000, and 0.1200 m. Make three trials for each amplitude and measure the time for 10 periods for each trial. Record all results in Data Table 2.

## Mass Variation

1. Place a hooked mass of 0.050 kg on the spring and let it hang at rest. Displace the mass 0.0500 m below the equilibrium ($A = 0.0500\,\text{m}$), release it, and let the system oscillate. Measure the time for 10 periods of the motion and record it in Data Table 3 as $\Delta t$. Repeat the procedure two more times for a total of three trials with this mass.
2. Repeat the procedure of Step 1 with the same $A$ for values of the mass $M$ of 0.100, 0.200, 0.300, 0.400, and 0.500 kg. Perform three trials of the time for 10 periods for each mass and record the results in Data Table 3.
3. Determine the mass of the spring $m_s$ and record it in Data Table 3.

# CALCULATIONS

## Spring Constant

1. Calculate the force $Mg$ for each mass and record the values in Data and Calculations Table 1. Use the value of $9.80\,\text{m/s}^2$ for $g$.
2. Perform a linear least squares fit to the data with $Mg$ as the vertical axis and $y$ as the horizontal axis. Record in Data and Calculations Table 1 the slope of the fit as the spring constant $k$ and the correlation coefficient $r$.

## Amplitude Variation

1. Calculate the mean $\overline{\Delta t}$ and standard error $\alpha_t$ of the three trials for each amplitude. Record the results in Calculations Table 2.
2. Calculate the period $T$ from $T = \overline{\Delta t}/10$. Record the results in Calculations Table 2.

## Mass Variation

1. Calculate the mean $\overline{\Delta t}$ and standard error $\alpha_t$ for the three trials for each mass. Record the results in Calculations Table 3.
2. Calculate the period $T$ from $T = \overline{\Delta t}/10$. Record the results in Calculations Table 3.
3. If both sides of Equation 4 are squared the result is

$$T^2 = \frac{4\pi^2}{k}(M + Cm_s) \qquad \text{(Eq. 5)}$$

4. Equation 5 states that $T^2$ is proportional to $M$ with $4\pi^2/k$ as the slope and $4\pi^2 Cm_s/k$ as the intercept. Calculate and record the values of $T^2$ in Calculations Table 3. Perform a linear least squares fit with $T^2$ as the vertical axis and $M$ as the horizontal axis. Record the values of the slope, intercept, and $r$ in Calculations Table 3.
5. Equate the value of the slope determined in Step 4 to $4\pi^2/k$ and solve for the value of $k$ in the resulting equation. Record this value of $k$ in Calculations Table 3.
6. Calculate the percentage difference between the value of $k$ determined in Step 5 and the value of $k$ determined earlier and record it in Calculations Table 3.
7. Equate the value of the intercept determined in Step 4 to $4\pi^2 Cm_s/k$ and solve for the value of $C$ in the resulting equation. In the equation, use the value of $k$ determined in Step 5. Record the value $C$ in Calculations Table 3.

# GRAPHS

1. Graph the data from Calculations Table 1 for force $Mg$ versus displacement $y$ with $Mg$ as the vertical axis and $y$ as the horizontal axis. Show on the graph the straight line obtained from the fit to the data.
2. Graph the data from Calculations Table 2 for the period $T$ versus the amplitude $A$ with $T$ as the vertical axis and $A$ as the horizontal axis.
3. Graph $T^2$ versus $M$ with $T^2$ as the vertical axis and $M$ as the horizontal axis. Also show on the graph the straight line obtained from the linear least squares fit to the data.

Name Section Date

20 **LABORATORY 20** *Simple Harmonic Motion—Mass on a Spring*

## PRE-LABORATORY ASSIGNMENT

1. Describe in words and give an equation for the kind of force that produces simple harmonic motion.

2. Other than the type of force that produces it, what characterizes simple harmonic motion?

3. A spring has a spring constant $k = 8.75$ N/m. If the spring is displaced 0.150 m from its equilibrium position, what is the force that the spring exerts? Show your work.

4. A spring of constant $k = 11.75$ N/m is hung vertically. A 0.500 kg mass is suspended from the spring. What is the displacement of the end of the spring due to the weight of the 0.500 kg mass? Show your work.

5. A spring with a mass on the end of it hangs in equilibrium a distance of 0.4200 m above the floor. The mass is pulled down a distance 0.0600 m below the original position, released, and allowed to oscillate. How high above the floor is the mass at the highest point in its oscillation? Show your work.

6. A massless spring has a spring constant of $k = 7.85$ N/m. A mass $M = 0.425$ kg is placed on the spring, and it is allowed to oscillate. What is the period $T$ of oscillation? Show your work.

7. A massless spring of $k = 6.45$ N/m has a mass $M = 0.300$ kg on the end of the spring. The mass is pulled down 0.0500 m and released. What is the period $T$ of the oscillation? What is the period $T$ if the mass is pulled down 0.1000 m and released? State clearly the reasoning for your answer.

Name ______ Section ______ Date ______

Lab Partners ______

20 LABORATORY 20 *Simple Harmonic Motion—Mass on a Spring*

## LABORATORY REPORT

*Data and Calculations Table 1*

| $M$ (kg) | $Mg$ (N) | $y$ (m) | $k$ (N/m) | $r$ |
|---|---|---|---|---|
| 0.050 | | | | |
| 0.100 | | | | |
| 0.200 | | | | |
| 0.300 | | | | |
| 0.400 | | | | |
| 0.500 | | | | |

*Data and Calculations Table 2* $y_o$ = ______ m

| $A$ (m) | $\Delta t_1$ (s) | $\Delta t_2$ (s) | $\Delta t_3$ (s) | $\overline{\Delta t}$ (s) | $\alpha_t$ (s) | $T$ (s) |
|---|---|---|---|---|---|---|
| 0.0200 | | | | | | |
| 0.0400 | | | | | | |
| 0.0600 | | | | | | |
| 0.0800 | | | | | | |
| 0.1000 | | | | | | |
| 0.1200 | | | | | | |

*Data and Calculations Table 3*

$m_s =$ _______________ kg

| $M$ (kg) | $\Delta t_1$ (s) | $\Delta t_2$ (s) | $\Delta t_3$ (s) | $\overline{\Delta t}$ (s) | $\alpha_t$ (s) | $T$ (s) | $T^2$ ($s^2$) |
|---|---|---|---|---|---|---|---|
| 0.050 | | | | | | | |
| 0.100 | | | | | | | |
| 0.200 | | | | | | | |
| 0.300 | | | | | | | |
| 0.400 | | | | | | | |
| 0.500 | | | | | | | |

| | | |
|---|---|---|
| Slope = | Intercept = | $r =$ |
| $k =$ N/m | $C =$ | % Diff = |

## SAMPLE CALCULATIONS

1. $Mg =$
2. $T = \overline{\Delta t}/10$
3. $T^2 =$
4. $k = 4\pi^2/(\text{Slope})$
5. $C = k(\text{Intercept})/(4\pi^2 m_s) =$

## QUESTIONS

1. Do the data for the displacement of the spring $y$ versus the applied force $Mg$ indicate that the spring constant is constant for this range of forces? State clearly the evidence for your answer.

2. How is the period $T$ expected to depend upon the amplitude $A$? State how your data do or do not confirm this expectation.

**3.** Consider the value you obtained for $C$. If you express that fraction as a whole number fraction, which of the following would best fit your data? (½ ⅓ ¼ ⅕)

**4.** Calculate $T$ predicted by Equation 3 for $M = 0.050$ kg. Calculate $T$ predicted by Equation 4 with the same $M$ and your value of $C$. What is the percentage difference between these two values of $T$? Do the same calculations for $M = 0.500$ kg. For which case are the percentage differences greater and why are they greater?

**5.** The determination of $T$ was done by measuring for 10 periods. Why was the time for more than one period measured? If there is an advantage to measuring for 10 periods, why not measure for 1000 periods?