The Pendulum—Approximate Simple Harmonic Motion

OBJECTIVES

- ❑ Investigate the dependence of the period T of a pendulum on the length L and the mass M of the bob.
- ❑ Demonstrate that the period T of a pendulum depends slightly on the angular amplitude of the oscillation for large angles, but that the dependence is negligible for small angular amplitude of oscillation.
- ❑ Determine an experimental value of the acceleration due to gravity g by comparing the measured period of a pendulum with the theoretical prediction.

EQUIPMENT LIST

- Pendulum clamp, string, and calibrated hooked masses, laboratory timer
- Protractor and meter stick

THEORY

A mass M moving in one dimension is said to exhibit **simple harmonic motion** if its displacement x from some equilibrium position is described by a single sine or cosine function. This happens when the particle is subjected to a force F directly proportional to the magnitude of the **displacement** and directed toward the equilibrium position. In equation form this is

$$F = -kx \qquad \text{(Eq. 1)}$$

The **period** T of the motion is the time for one complete oscillation, and it is determined by the mass M and the constant k. The equation that describes the dependence of T on M and k is

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad \text{(Eq. 2)}$$

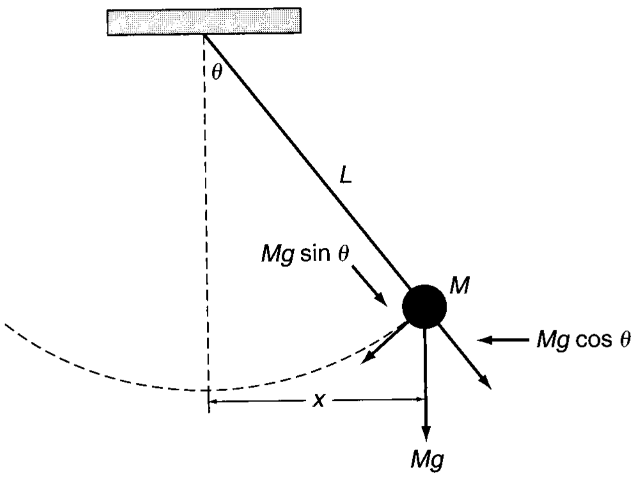

Figure 19-1 Force components acting on the mass bob of a simple pendulum.

A pendulum does not exactly satisfy the conditions for simple harmonic motion, but it approximates them under certain conditions. An ideal pendulum is a point mass M on one end of a massless string with the other end fixed as shown in Figure 19-1. The motion of the system takes place in a vertical plane when the mass M is released from an initial angle θ with respect to the vertical.

The downward weight of the pendulum can be resolved into two components as shown in Figure 19-1. The component $Mg\cos\theta$ equals the magnitude of the tension N in the string. The component $Mg\sin\theta$ acts tangent to the arc along which the mass M moves. This component provides the force that drives the system. In equation form the force F along the direction of motion is

$$F = -Mg\sin\theta \qquad \text{(Eq. 3)}$$

For small values of the initial angle θ, we can use the approximation $\sin\theta \approx \tan\theta \approx x/L$ in Equation 3, which gives

$$F = -\frac{Mg}{L}x \qquad \text{(Eq. 4)}$$

Although Equation 4 is an approximation, it is of the form of Equation 1 with $k = Mg/L$. Using that value of k in Equation 2 gives

$$T = 2\pi\sqrt{\frac{M}{Mg/L}} = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(Eq. 5)}$$

Equation 5 predicts that the period T of a simple pendulum is independent of the mass M and the angular amplitude θ and depends only on the length L of the pendulum.

The exact solution to the period of a simple pendulum without making the small angle approximation leads to an infinite series of terms, with each successive term becoming smaller. Equation 6 gives the first three terms in the series. They are sufficient to determine the very slight dependence of the period T on the angular amplitude of the motion.

$$T = 2\pi\sqrt{\frac{L}{g}}\left[1 + 1/4\sin^2(\theta/2) + 9/64\sin^4(\theta/2) + \cdots\right] \qquad \text{(Eq. 6)}$$

For an ideal pendulum with no friction, the motion repeats indefinitely with no reduction in the amplitude as time goes on. For a real pendulum there will always be some friction, and the amplitude of the motion decreases slowly with time. However, for small initial amplitudes, the change in the period as the amplitude decreases is negligible. This fact is the basis for the pendulum clock. Pendulum clocks, in one form or another, have been used for more than 300 years. For more than 100 years, clockmakers have built extremely accurate clocks by successfully employing devices to compensate for small changes in the length of the pendulum caused by temperature variations.

EXPERIMENTAL PROCEDURE

Length

1. The dependence of T on the length of the pendulum will be determined with a fixed mass and fixed angular amplitude. Place a 0.2000 kg hooked calibrated mass on a string with a loop in one end. Adjust the position at which the other end of the string is clamped in the pendulum clamp until the distance from the point of support to the center of mass of the hooked mass is 1.0000 m. The length L of each pendulum is from the point of support to the center of mass of the bob. The center of mass of the hooked masses will usually not be in the center because the hooked masses are not solid at the bottom. Estimate how much this tends to raise the position of the center of mass and mark the estimated center of mass on each hooked mass.
2. Displace the pendulum 5.0° from the vertical and release it. Measure the time Δt for 10 complete periods of motion and record that value in Data Table 1. It is best to set the pendulum in motion, and then begin the timer as it reaches the maximum displacement, counting 10 round trips back to that position. Repeat this process two more times for a total of three trials with this same length. The pendulum should move in a plane as it swings. If the mass moves in an elliptical path, it will lead to error.
3. Repeat the procedure of Step 2, using the same mass and an angle of 5.0° for pendulum lengths of 0.8000, 0.6000, 0.5000, 0.3000, 0.2000, and 0.1000 meters. Do three trials at each length. The length of the pendulum is from the point of support to the center of mass of the hooked mass.

Mass

1. The dependence of T on M will be determined with the length L and amplitude θ held constant. Place a 0.0500 kg mass on the end of the string and adjust the point of support of the string until the pendulum length is 1.0000 m. Displace the mass 5.0° and release it. Measure the time Δt for 10 complete periods of the motion and record it in Data Table 2. Repeat the procedure two more times for a total of three trials.
2. Keep the length constant at $L = 1.0000$ m and repeat the procedure above for M of 0.1000, 0.2000, and 0.5000 kg. Because L is from the point of support to the center of mass, you will need to make slight adjustments in the string length to keep L constant for the different masses.

Amplitude

1. The dependence of T on amplitude of the motion will be determined with L and M constant. Construct a pendulum with $L = 1.000$ m and $M = 0.200$ kg. Measure the time Δt for 10 complete periods with amplitude 5.0°. Repeat two more times for a total of three trials at this amplitude. Record all results in Data Table 3.
2. Repeat the procedure above for amplitudes of 10.0°, 20.0°, 30.0°, and 45.0°. Do three trials for each amplitude and record the results in Data Table 3.

CALCULATIONS

Length

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t of the three trials for each of the lengths. Record those results in Calculations Table 1.

2. Calculate the period T from $T = \overline{\Delta t}/10$ and record it in Calculations Table 1.
3. According to Equation 5 the period T should be proportional to $\sqrt{L}$. For each of the values of L calculate $\sqrt{L}$ and record the results in Calculations Table 1. Perform a linear least squares fit with T as the vertical axis and $\sqrt{L}$ as the horizontal axis. By Equation 5 the slope of this fit should equal $2\pi/\sqrt{g}$. Equate the slope determined from the fit to $2\pi/\sqrt{g}$, treating g as unknown. Solve this equation for g and record that value as g_{exp} in Calculations Table 1. Also record the value of the correlation coefficient r for the fit.

Mass

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t of the three trials for each of the masses. Record those results in Calculations Table 2.
2. Calculate the period T from $T = \overline{\Delta t}/10$ and record in Calculations Table 2.

Amplitude

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t for the three trials at each amplitude. Record the results in Calculations Table 3.
2. Calculate $T = \overline{\Delta t}/10$ and record the results in Calculations Table 3 as T_{exp}.
3. Equation 6 is the theoretical prediction for how the period T should depend on the amplitude. Use $L = 1.000$ m and $M = 0.200$ kg in Equation 6 to calculate the T predicted for the values of θ. Record them in Calculations Table 3 as T_{theo}.
4. For the experimental values of the period T_{exp} calculate the ratio of the period at the other angles to the period at $\theta = 5.0°$. Call this ratio $\left(T_{exp}(\theta)/T_{exp}(5.0°)\right)$. Record these values in Calculations Table 3.
5. For the theoretical values of the period T_{theo} calculate the ratio of the period at the other angles to the period at $\theta = 5.0°$. Call this ratio $\left(T_{theo}(\theta)/T_{theo}(5.0°)\right)$. Record these values in Calculations Table 3.

GRAPHS

1. Consider the data for the dependence of the period T on the length L. Graph the period T as the vertical axis and $\sqrt{L}$ as the horizontal axis. Also show on the graph the straight line obtained by the linear fit to the data.
2. Consider the data for the dependence of the period T on the mass M. Graph the period T as the vertical axis and the mass M as the horizontal axis.

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PRE-LABORATORY ASSIGNMENT

1. What is the requirement for a force to produce simple harmonic motion?

2. A particle of mass $M = 1.35$ kg is subject to a force $F = -0.850\ x$, where x is the displacement of the particle from equilibrium. The units of force F are Newtons, and the units of x are meters. What is the period T of its motion? Show your work.

3. A simple pendulum of length $L = 0.800$ m has a mass $M = 0.250$ kg. What is the tension in the string when it is at an angle $\theta = 12.5°$? Show your work.

4. In Question 3, what is the component of the weight of M that is directed along the arc of the motion of M? Show your work.

5. What is the period T of the motion of the pendulum in Question 3? Assume that the period is independent of θ. Show your work.

6. What would be the period T of the pendulum in Question 3 if L was unchanged but $M = 0.500$ kg? Assume that the period is independent of θ. Show your work.

7. Determine the period T of the pendulum in Question 3 if everything else stays the same, but $\theta = 45.0°$. Do not assume that the period is independent of θ. Show your work.

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LABORATORY REPORT

Data and Calculations Table 1

L (m)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	$\overline{\Delta t}$ (s)	α_t (s)	T (s)	$\sqrt{L}$ ($\sqrt{\text{m}}$)
1.0000							
0.8000							
0.6000							
0.5000							
0.3000							
0.2000							
0.1000							
Slope =			g_{exp} =		m/s^2	r =	

Data and Calculations Table 2

M (kg)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	$\overline{\Delta t}$ (s)	α_t (s)	T (s)
0.0500						
0.1000						
0.2000						
0.5000						

Data and Calculations Table 3

θ	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	$\overline{\Delta t}$ (s)	α_t (s)	T_{exp} (s)	T_{theo} (s)	$\frac{T_{exp}(\theta)}{T_{exp}(5°)}$	$\frac{T_{theo}(\theta)}{T_{theo}(5°)}$
5.0°									
10.0°									
20.0°									
30.0°									
45.0°									

SAMPLE CALCULATIONS

1. $T_{exp} = \overline{\Delta t}/10 =$
2. $\sqrt{L} =$
3. $g_{exp} = 4\pi^2/(\text{slope})^2 =$
4. $T_{theo} =$

QUESTIONS

1. In general, what is the precision of the measurements of T? Answer this question by considering what percentage is α_t of $\overline{\Delta t}$ for the measurements as a whole.

2. Do your data confirm the expected dependence of the period T on the length L of a pendulum? Consider the correlation coefficient r for the least squares fit in your answer.

3. Comment on the accuracy of your experimental value for the acceleration due to gravity g.

4. What does the theory predict for the shape of the graph of period T versus M? Do your data confirm this expectation? Calculate the mean and standard error of the periods for the four masses and comment on how this relates to mass independence of T.

5. Do your measured values for the period T as a function of the amplitude θ confirm the theoretical predictions? State clearly what is expected and what your data show.

6. The values of T were determined by measuring the time for 10 periods. Why is the time for more than one period measured? If there is an advantage to measuring for 10 periods, why not measure for 1000 periods?