The Ballistic Pendulum and Projectile Motion

OBJECTIVES

- ❑ Investigate how the initial velocity of a ball fired into a ballistic pendulum is related to the initial velocity with which the pendulum plus ball moves after the collision.
- ❑ Investigate the kinetic energy loss in the collision of the ball with the pendulum.
- ❑ Determine the initial velocity of the ball by firing it as a projectile and compare it with the velocity determined by the collision.

EQUIPMENT LIST

- Ballistic pendulum apparatus with projectile ball
- Laboratory balance and calibrated masses
- Meter stick, plain paper, carbon paper, and masking tape

THEORY

Ballistic Pendulum

The principle of **conservation of momentum** states that the total momentum of a system of particles remains constant if there are no external forces acting on the system. Collision processes are good examples of this concept. In this laboratory we will use a ballistic pendulum to measure the velocity of a ball projected by a spring gun. Figure 13-1 shows a ball of mass m moving initially in the horizontal direction with speed v_{xo} that then strikes a pendulum designed to catch the ball. The pendulum of mass M catches the ball and swings about pivot point O to some maximum height y_2 above its original height y_1. The system of ball plus pendulum rises a vertical distance of $y_2 - y_1$ as a result of the process.

Momentum is conserved because the only forces acting on the ball and the pendulum in the direction of motion are the forces of the collision. The two particles stick together after the collision and move with the same velocity V. A collision where particles stick together is called a **completely inelastic collision.** The equation for conservation of momentum is

$$mv_{xo} = (m + M)V \qquad \text{(Eq. 1)}$$

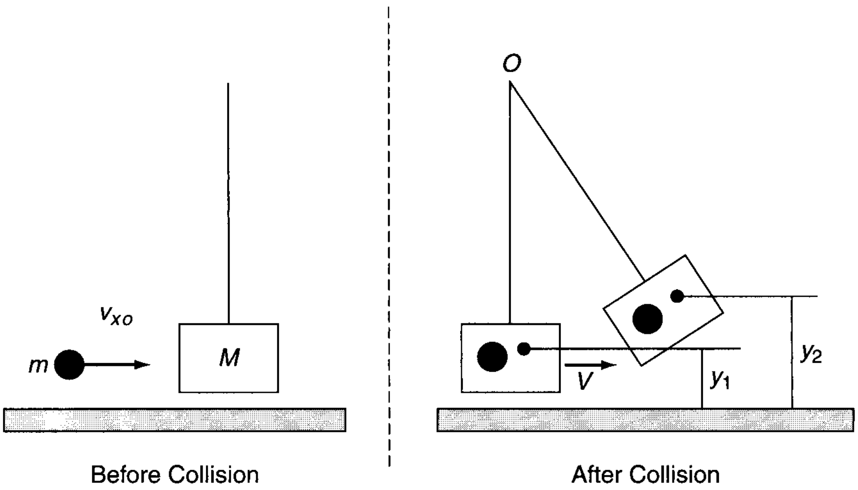

Figure 13-1 Ballistic pendulum of mass M before and after collision with ball of mass m.

The collision does not conserve **mechanical energy,** but mechanical energy is conserved as the ball plus pendulum swings up along the arc. The **kinetic energy** immediately after the collision is converted into **gravitational potential energy.** In equation form

$$\tfrac{1}{2}(m+M)V^2 = (m+M)g(y_2 - y_1) \qquad \text{(Eq. 2)}$$

Solving for V gives

$$V = \sqrt{2g(y_2 - y_1)} \qquad \text{(Eq. 3)}$$

Solving Equation 1 for the initial velocity of the ball gives

$$v_{xo} = \left(\frac{m+M}{m}\right)V \qquad \text{(Eq. 4)}$$

A measurement of $y_2 - y_1$ used in Equation 3 gives a value of V to be used in Equation 4 to determine v_{xo}.

Projectile Motion

If the pendulum is raised and the pawl is placed in one of the notches on the track, the ball can now travel a horizontal distance X while it falls a vertical distance Y as shown in Figure 13-2.

The original velocity of the ball is completely in the x direction with no y component. The acceleration due to gravity in the y direction is the only acceleration of the ball. The horizontal displacement X and the vertical displacement Y as a function of time t after the ball is launched are

$$X = v_{xo}\, t \qquad \text{(Eq. 5)}$$

$$Y = \tfrac{1}{2}gt^2 \qquad \text{(Eq. 6)}$$

Equation 6 has been written with positive displacements down in the same direction as g. Combining Equations 5 and 6 to eliminate time t gives v_{xo} in terms of X and Y as

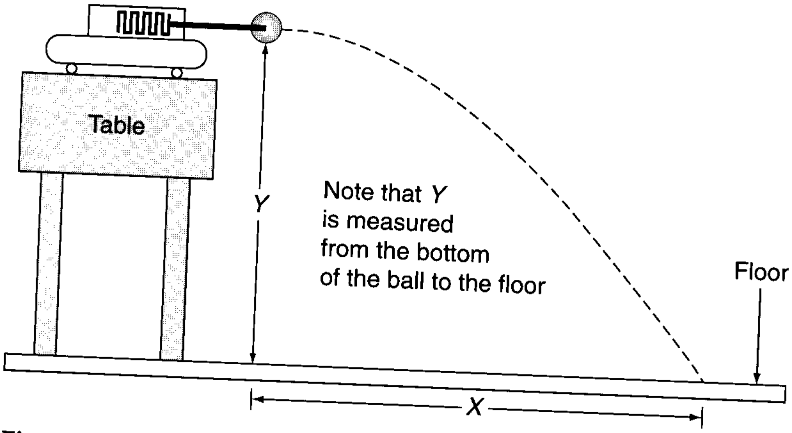

Figure 13-2 Motion of the ball moving horizontal distance X while free falling height Y.

$$v_{xo} = \frac{X}{\sqrt{2Y/g}} \tag{Eq. 7}$$

Equation 7 can be used to determine the initial velocity v_{xo} by firing the projectile from a known height Y and measuring the value of X that results.

The velocity in the y direction is initially zero. As the projectile falls under the influence of gravity, it acquires a velocity in the y direction given by

$$V_y = gt = g\sqrt{2Y/g} = \sqrt{2gY} \tag{Eq. 8}$$

EXPERIMENTAL PROCEDURE

Ballistic Pendulum

1. Slide the projectile ball (which has a hole in it) onto the rod of the spring gun (Figure 13-3). When the ball is in the pendulum, be careful when removing it. The spring that catches the ball in the pendulum can be easily broken. With the ball on the rod, cock the gun by pushing against the ball until the latch

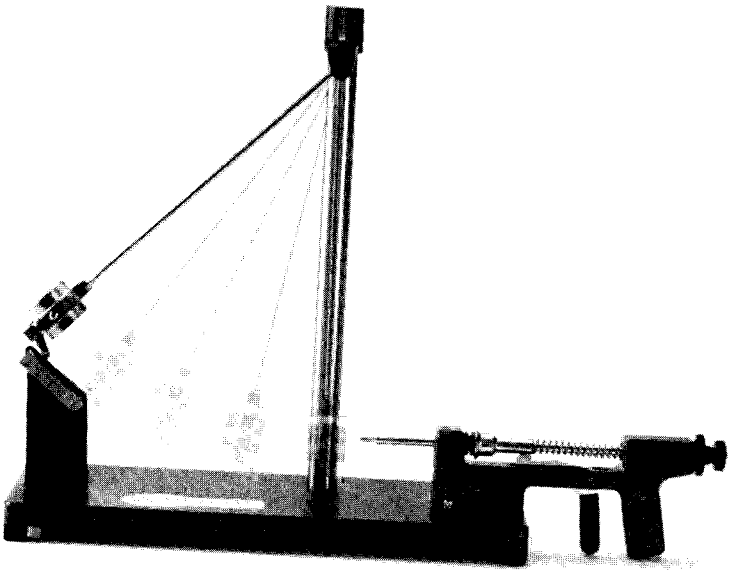

Figure 13-3 Ballistic pendulum apparatus. (Photo courtesy of Central Scientific Co., Inc.)

catches. *Be very careful not to get your hand caught in the spring gun mechanism.* Fire the gun several times to see how it operates. There are two common problems. If the ball does not catch in the pendulum bob, the spring in the bob should be adjusted or replaced. If the pawl that is designed to catch on the notched track does not engage, the pendulum suspension should be adjusted by means of the screws at the suspension points.

2. A sharp curved point on the side of the pendulum (or on some models a dot) marks the center of mass of the pendulum-ball system. Let the pendulum bob hang vertically and measure the distance y_1, the center of mass point above the base of the gun (Figure 13-1). Record the value of y_1 in Data Table 1.
3. Fire the ball into the stationary pendulum while it hangs freely at rest. The pendulum will catch the ball, swing up, and then lodge in the notched track. Record in Data Table 1 the position number, p, at which the pawl on the pendulum catches on the track. Measure the distance y_2 of the center of mass point above the base of the apparatus (Figure 13-1) and record it in Data Table 1. Repeat this procedure four more times for a total of five trials, recording the position p and measuring the distance y_2 for each trial.
4. Loosen the screws holding the pendulum in its support and remove the pendulum consisting of the rod and the bob. Determine the mass of the pendulum (bob and rod) using a laboratory balance. Record the pendulum mass as M in Data Table 1. Determine the projectile ball mass and record it in Data Table 1 as m.

Projectile Motion

In the following procedure, be extremely careful not to fire the ball when anyone is in a position to be struck by the ball. Serious injury could result.

1. Raise the pendulum and secure it so that the ball can be fired under it.
2. Place the apparatus near the front edge of the laboratory table so that the ball will strike the floor before it strikes a wall or any other object. The gun must be fired each time from the same position relative to the table. It may be necessary to clamp the apparatus to the table. Place a piece of heavy cardboard or some other object in a position to catch the ball after it strikes the floor but before it strikes a wall. *Do not allow the ball to strike a wall because it will likely damage it.* Make several test firings to locate the approximate place where the ball will land on the floor.
3. Place a sheet of white paper on the floor approximately centered where test firings have landed. Place a piece of carbon paper over the white paper so that the ball striking the carbon paper will leave a dot on the white paper. Tape both of the papers to the floor.
4. Place the ball on the rod of the spring gun. The vertical distance Y that the ball will fall is the distance from the *bottom* of the ball to the floor as shown in Figure 13-2. Measure this distance to the nearest 0.1 mm and record it in Data Table 2 as Y.
5. Fire the ball five times onto the same sheet of paper. Place the ball on the rod and measure the horizontal distance X to the nearest 0.1 mm from the center of the ball to the center of each dot on the paper. Record these five values of X in Data Table 2.

CALCULATIONS

Ballistic Pendulum

1. Calculate the distance $y_2 - y_1$ that the ball plus pendulum rises for each trial. Record these values and all other calculations in this section in Calculations Table 1.
2. From Equation 3 calculate the velocity V for each of the five trials.
3. From Equation 4 calculate the initial speed v_{xo} for the five trials.
4. Calculate the mean $\bar{v}_{xo}$ and the standard error α_v of the five values of v_{xo}.

Projectile Motion

1. Use Equation 7 to calculate the value of v_{xo} for each of the five values of X. Use a value of $g = 9.80\,\text{m/s}^2$. Record these values in Calculations Table 2.
2. Calculate the mean $\bar{v}_{xo}$ and the standard error α_v of the five values of v_{xo}. Record them in Calculations Table 2.

Name Section Date

LABORATORY 13 *The Ballistic Pendulum and Projectile Motion*

PRE-LABORATORY ASSIGNMENT

1. What are the conditions under which the total momentum of a system of particles is conserved?

2. What kind of collision conserves kinetic energy?

3. What kind of collision does not conserve kinetic energy? What kind of collision results in the maximum loss of kinetic energy?

4. A ball of mass 0.075 kg is fired horizontally into a ballistic pendulum as shown in Figure 13-1. The pendulum mass is 0.350 kg. The ball is caught in the pendulum, and the center of mass of the system rises a vertical distance of 0.145 m in the earth's gravitational field. What was the original speed of the ball? Assume that $g = 9.80\,\text{m/s}^2$. Show your work.

5. How much kinetic energy was lost in the collision of Question 4? Show your work.

6. A projectile is fired in the earth's gravitational field with a horizontal velocity of $v = 9.00$ m/s. How far does it go in the horizontal direction in 0.550 s? Show your work.

7. How far does the projectile of Question 6 fall in the vertical direction in 0.550 s? Show your work.

8. A projectile is launched in the horizontal direction. It travels 2.050 m horizontally while it falls 0.450 m vertically, and it then strikes the floor. How long is the projectile in the air? Show your work.

9. What was the original velocity of the projectile described in Question 8? Show your work.

Name _______________ Section _______ Date _______

Lab Partners _______________

LABORATORY 13 The Ballistic Pendulum and Projectile Motion

LABORATORY REPORT

Data Table 1

Trial	p	y_2 (m)
1		
2		
3		
4		
5		
$m =$ kg	$M =$ kg	$y_1 =$ m

Calculations Table 1

$y_2 - y_1$ (m)	V (m/s)	v_{xo} (m/s)
$\overline{v}_{xo} =$ m/s	$\alpha_v =$ m/s	

Data Table 2

Trial	X (m)
1	
2	
3	
4	
5	
$Y =$ m	

Calculations Table 2

v_{xo} (m/s)	
$\overline{v}_{xo} =$ m/s	$\alpha_v =$ m/s

SAMPLE CALCULATIONS

1. $y_2 - y_1 =$

2. $V = \sqrt{2g(y_2 - y_1)} =$

3. (Ballistic pendulum) $v_{xo} = \left(\dfrac{m + M}{m}\right) V$

4. (Projectile motion) $= v_{xo} = \dfrac{X}{\sqrt{2Y/g}}$

QUESTIONS

1. Compare the two different values of $\bar{v}_{xo}$. Calculate the difference and the percentage difference between them. State whether the two measurements agree within the combined standard errors of the two values of $\bar{v}_{xo}$.

2. Can you make any statement about the accuracy of the two values of $\bar{v}_{xo}$? Are either of these values more precise than the other? State clearly the basis for your answer in each case.

3. Calculate the loss in kinetic energy when the ball collides with the pendulum as the difference between $\frac{1}{2} m v_{xo}^2$ (the kinetic energy before) and $\frac{1}{2}(m + M)V^2$ (the kinetic energy immediately after the collision).

4. What is the fractional loss in kinetic energy? Calculate by dividing the loss calculated in Question 3 by the original kinetic energy.

5. Calculate the ratio $M/(m+M)$ for the values of m and M in Data Table 1. Compare this ratio with the ratio calculated in Question 4. Express the fractional loss of kinetic energy in symbol form and use equations from the lab to show it should equal $M/(m+M)$.

6. It was assumed that the ball fired as a projectile moved exactly in the horizontal direction. If it moved at some small angle θ to the horizontal, the correct equation would be $(4.90X^2/v_o^2)\tan^2\theta - X\tan\theta + (Y + 4.90X^2/v_o^2) = 0$ with the initial velocity labeled v_o. Use the value of v_o from the ballistic pendulum measurement and the measured X and Y in the equation and solve for the angle θ. If the ball was fired at angle θ to the horizontal it would account for the difference in the two measured values of v_o. The equation is a quadratic in $\tan\theta$ and Y is negative in the equation. Is the θ you found small enough that it is plausible that the projectile might deviate that much from horizontal?