Newton's Second Law on Atwood's Machine

OBJECTIVES

- ❑ Investigate the acceleration produced by a series of different forces applied to a fixed mass, and demonstrate that the acceleration is proportional to the applied force.
- ❑ Demonstrate that the constant of proportionality between the acceleration and the applied force is the mass to which the force is applied.
- ❑ Determine the frictional force that acts on the system.

EQUIPMENT LIST

- Atwood's machine pulley (very low-friction ball-bearing pulley)
- Calibrated slotted masses, slotted mass holders, laboratory balance
- Laboratory timer (capable of measuring to 0.01 s), strong thin nylon string, and meter stick

THEORY

The relationship between the *net* force F exerted on a mass m and the acceleration a of the mass is **Newton's Second Law.**

$$F = ma \qquad \text{(Eq. 1)}$$

The system shown in Figure 9-1 is called an Atwood's machine. It consists of two masses at the ends of a string passing over a pulley. Also shown in the figure is a free-body diagram of the forces. For $m_2 > m_1$, Equation 1 applied to each mass gives

$$T - m_1 g = m_1 a \quad \textit{and} \quad m_2 g - T = m_2 a \qquad \text{(Eq. 2)}$$

where T is the tension in the string, and a is the magnitude of the acceleration of either mass. Combining Equations 2 leads to

$$(m_2 - m_1)g = (m_1 + m_2)a \qquad \text{(Eq. 3)}$$

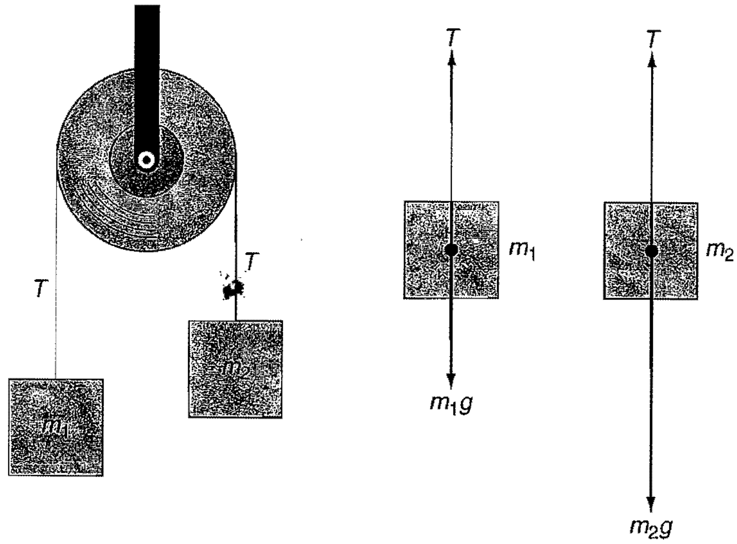

Figure 9-1 Atwood's machine and free-body diagram of the forces on each mass.

Equation 3 states that a force $(m_2 - m_1)g$ equal to the difference in the weight of the two masses acts on the sum of the masses $(m_1 + m_2)$ to produce an acceleration a of the system. There will be a frictional force f in the system that opposes the applied force $(m_2 - m_1)g$. Including the frictional force but moving it to the other side of the equation gives

$$(m_2 - m_1)g = (m_1 + m_2)a + f \qquad \text{(Eq. 4)}$$

An Atwood's machine is shown in Figure 9-2 where $m_2 > m_1$, the mass m_1 is initially on the floor, and m_2 is released from rest at distance x above the floor at $t = 0$. Successive positions of the two masses are shown in Figure 9-2 at later times until the final picture shows m_2 as it strikes the floor at some time t after its release. The relationship between the distance x, the acceleration a of the system, and the time t is

$$x = \frac{at^2}{2} \qquad \text{(Eq. 5)}$$

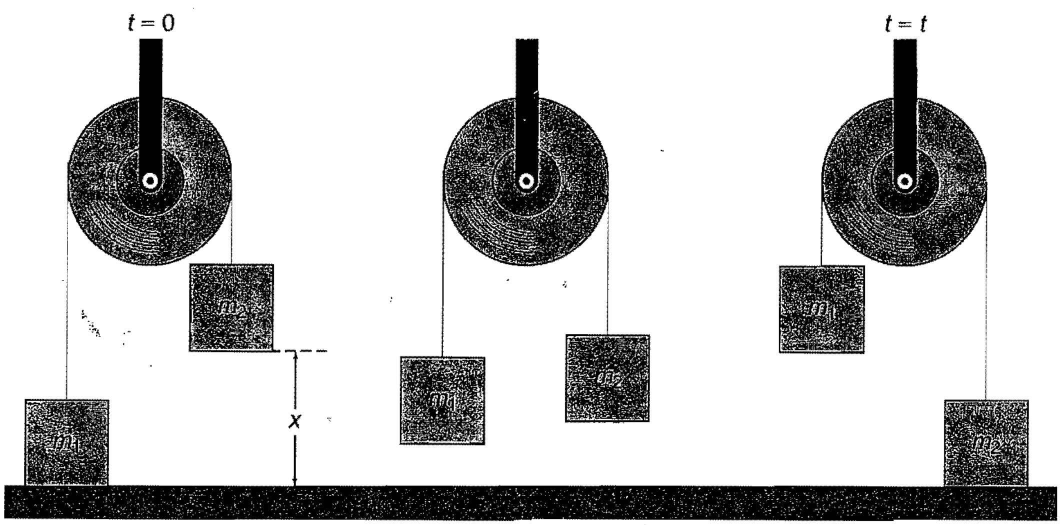

Figure 9-2 Atwood's machine as mass m_2 falls a distance x in time t.

Solving Equation 5 for a in terms of the measured quantities x and t gives

$$a = \frac{2x}{t^2} \qquad \text{(Eq. 6)}$$

This laboratory will measure the acceleration for the Atwood's machine for several different values of the applied force $(m_2 - m_1)g$ using a fixed total mass $(m_1 + m_2)$. Because the pulley is not massless, some portion of its mass should be included in the total mass. You will be challenged to discover what fraction of the pulley's mass should be included when you analyze the data that you will take in the laboratory.

EXPERIMENTAL PROCEDURE

1. Place 1.0000 kg of slotted masses (including a 0.0500 kg mass holder) on each pan of a laboratory balance. Include five 0.0020 kg masses on one of the pans. If all the masses are accurate, the scale should be balanced. If the scales are not exactly balanced, add 0.0010 kg slotted masses to whichever side is needed to produce as perfect a balance as can be obtained. The purpose of this procedure is to produce two masses that are as nearly equal as possible. Record in the Data Table the sum of all mass on the balance as $(m_1 + m_2)$. Record in the Data Table the mass of the pulley as m_p.
2. Place these two collections of slotted masses on mass holders at each end of a string over the pulley. Place the group of masses that contain the collection of 0.0020 kg masses on the left side (m_1) of the pulley and the other masses on the right (m_2).
3. Transfer a 0.0060 kg mass from the left side (m_1) to the right side (m_2) while holding the system fixed. This produces a mass difference $(m_2 - m_1)$ of 0.0120 kg.
4. While one partner holds mass m_1 on the floor, another partner should measure the distance x from the bottom of mass m_2 to the floor as shown in Figure 9-2. The height of the pulley above the floor should be chosen as high as feasible but at least 1.000 m. Record this distance x in the Data Table. *Use this same distance for all measurements.*
5. Release mass m_1 and simultaneously start the timer. Stop the timer when mass m_2 strikes the floor. Repeat this measurement four more times for a total of five trials with a mass difference $(m_2 - m_1)$ of 0.0120 kg. Record all times in the Data Table.
6. Repeat Steps 4 and 5 using mass differences $(m_2 - m_1)$ of 0.0160, 0.0200, 0.0240, 0.0280, and 0.0320 kg by transferring a mass of 0.0020 kg each time. Make a total of five trials for each mass difference and record the measured times in the Data Table.

CALCULATIONS

1. Calculate and record the forces $(m_2 - m_1)g$ using $g = 9.800\,\mathrm{m/s^2}$.
2. Calculate the mean time $\bar{t}$ and the standard error α_t for the five measurements of time at each of the mass differences $(m_2 - m_1)$. Record those values in the Calculations Table.
3. Use Equation 6 to calculate the acceleration a from x and $\bar{t}$ for each value of applied force. Record these values of a in the Calculations Table.
4. Perform a linear least squares fit with the applied force $(m_2 - m_1)g$ as the vertical axis and the acceleration a as the horizontal axis. Record in the Calculations Table the slope of the fit as $(m_1+m_2)_{exp}$, the intercept of the fit as f, and the value of the correlation coefficient r.

GRAPHS

1. Make a graph of the data with the applied force as the vertical axis and the acceleration as the horizontal axis. Also show on the graph the straight line obtained by the least squares fit to the data.

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PRE-LABORATORY ASSIGNMENT

1. A net force of 3.50 N acts on a 2.75 kg object. What is the acceleration of the object? Show your work.

2. Describe the basic concept of the Atwood's machine. What is the net applied force? What is the mass to which this net force is applied? Show your work.

3. An Atwood's machine consists of a 1.060 kg mass and a 1.000 kg mass connected by a string over a massless and frictionless pulley. Use Equation 3 to find the acceleration of the system. Assume that g is 9.80 m/s^2. Show your work.

4. Suppose that the system in Question 3 has a frictional force of 0.056 N. Use Equation 4 to determine the acceleration of the system. Show your work.

The following data were taken with an Atwood's machine for which the total mass $m_1 + m_2$ is kept constant. For each of the values of mass difference $(m_2 - m_1)$ shown in the table, the time for the system to move $x = 1.000$ m was determined.

$(m_2 - m_1)$ (kg)	0.010	0.020	0.030	0.040	0.050
t (s)	8.30	5.06	3.97	3.37	2.98
a (m/s^2)					
$(m_2 - m_1)g$ (N)					

5. From the data above for x and time t, use Equation 6 to calculate the acceleration for each of the applied forces and record them in the table above. Show the calculation for the 0.010 kg mass difference as a sample calculation.

6. From the mass differences $(m_2 - m_1)$ calculate the applied forces $(m_2 - m_1)g$ and record them in the table above. Use a value of 9.80 m/s^2 for g. Show the calculation for the 0.010 kg mass difference as a sample calculation.

7. Perform a linear least squares fit with the applied force as the vertical axis and the acceleration as the horizontal axis. The slope of the fit is equal to the total mass $(m_1 + m_2)_{exp}$ and the intercept is the frictional force f. Record those and the value of the correlation coefficient r. *(This is the calculation that will be performed for the data of the laboratory.)*

$(m_1 + m_2)_{exp}$ = ________kg f = ________N r = ____________

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Lab Partners

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LABORATORY REPORT

Data Table

$(m_1 + m_2) =$ kg		$m_p =$ kg		$x =$ m	
$(m_2 - m_1)$ (kg)	t_1 (s)	t_2 (s)	t_3 (s)	t_4 (s)	t_5 (s)
0.012					
0.016					
0.020					
0.024					
0.028					
0.032					

Calculations Table

$(m_2 - m_1)g$ (N)	$\bar{t}$ (s)	α_t (s)	a (m/s^2)
$(m_1+m_2)_{exp} =$ kg	$f =$ N	$r =$	

SAMPLE CALCULATIONS

1. % Error = (E – K)/K × 100% =

QUESTIONS

1. According to statistical theory for six data points there is only 1% probability (1 chance in 100) that a value of $r \geq 0.917$ and a 0.1% probability (1 chance in 1000) that a value of $r \geq 0.974$ would be obtained for data that are uncorrelated. Based on this idea, what does your value of r indicate for the level of correlation of your data?

2. Divide the frictional force by the applied force, $(m_2 - m_1)g$, for each applied force and express it as a percentage in the space below. Friction would not be important if these percentages were a few percent, would be small if they were about 10%, and would be very important if they were 25% or greater. Make the best possible statement about the importance of friction for your data.

3. Your value of $(m_1 + m_2)_{exp}$ should be greater than your recorded value of $(m_1 + m_2)$ because of the effect of the pulley. Perform the calculations needed to determine what fraction of the pulley's mass appears to be included in your value of $(m_1 + m_2)_{exp}$. If you were to express that fraction as a whole number fraction, which of the following would best fit your data? (½ ⅓ ¼ ⅕)

4. The laboratory instructed you to transfer mass from one side of the pulley to the other side. Why was this procedure used instead of just adding mass to one side of the pulley to produce a larger force?

5. What concept is this laboratory designed to investigate? Describe the extent to which your data and analysis validate the concept.