Name ______________________ Section ____________ Date ____________

Laboratory 9

Newton's Second Law on the Atwood Machine

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. A net force of 3.50 N acts on a 2.75-kg object. What is the acceleration of the object?

2. Describe the basic concept of the Atwood machine. What is the net applied force? What is the mass to which this net force is applied?

3. An Atwood machine consists of a 1.0600-kg mass and a 1.0000-kg mass connected by a string over a massless and frictionless pulley. What is the acceleration of the system? Assume g is 9.80 m/s^2. Show your work.

4. Suppose the system in question 3 has a frictional force of 0.056 N. What is the acceleration of the system? Show your work.

5. Suppose the frictionless system described in question 3 is released from rest at $t = 0$. How long does it take for the large mass to fall 1.200 m? Show your work.

6. Suppose the system with friction described in question 4 is released from rest at $t = 0$. How long does it take for the large mass to fall 1.200 m? Show your work.

The following data were taken with an Atwood machine whose total mass $m_1 + m_2$ is kept constant. For each of the values of mass difference $m_2 - m_1$ shown in the table the time for the system to move $x = 1.000$ m was determined.

$m_2 - m_1$ (kg)	0.0100	0.0200	0.0300	0.0400	0.0500
t (s)	8.30	5.06	3.97	3.37	2.98
a (m/s^2)					
$(m_2 - m_1)g$ (N)					

7. From the data above for x and time t, calculate the acceleration for each of the applied forces and record them in the table above. Show a sample calculation.

8. From the mass differences $m_2 - m_1$ calculate the applied forces $(m_2 - m_1)g$ and record them in the table above. Use a value of 9.80 m/s^2 for g. Show a sample calculation.

9. Perform a linear least squares fit to the data with the applied force as the ordinate and the acceleration as the abscissa. Calculate the slope of the fit that is equal to the total mass $m_1 + m_2$, the intercept of the fit that is the frictional force f, and the regression coefficient of the fit r. Record the results below.

$m_1 + m_2 =$ ________ kg $\quad f =$ ________ N $\quad r =$ ________

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OBJECTIVES

When a net force F acts on a mass of m an acceleration a is produced. The relation between these quantities is given by Newton's second law as $F = ma$. In this laboratory an Atwood machine will be used to apply different forces to a fixed total mass to accomplish the following objectives:

1. Determination of the acceleration produced by a series of different forces applied to a given fixed mass
2. Demonstration that the acceleration of the mass is proportional to the magnitude of the applied force
3. Demonstration that the constant of proportionality between the acceleration and the applied force is the mass to which the force is applied
4. Determination of the frictional force that acts on the system

EQUIPMENT LIST

1. Atwood machine pulley (very-low-friction ball-bearing pulley)
2. Calibrated slotted masses and slotted-mass holders
3. Strong, thin nylon string
4. Laboratory timer, laboratory balance, and meter stick

THEORY

The relationship between the *net* force F exerted on a body of mass m and the acceleration a of the body was stated by Newton to be

$$F = ma \tag{1}$$

It is important to note that equation 1 is a vector equation, and thus it states that each component of the acceleration is determined by the net force in the direction of that component.

Consider the system shown in Figure 9.1, which is called an Atwood machine. It consists of two masses at the ends of a string passing over a pulley.

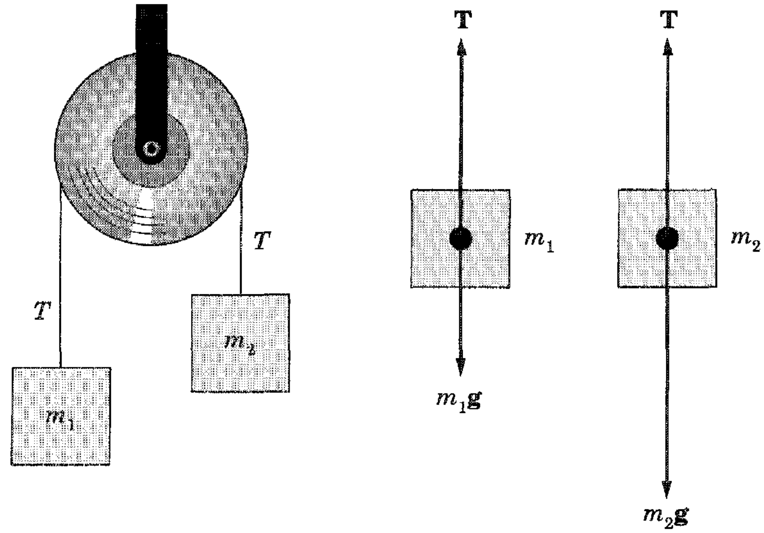

Figure 9.1 Atwood machine and free-body diagram of the forces on each mass.

Also shown in the figure is a free-body diagram of the forces on each mass. Assuming that $m_2 > m_1$, equation 1 applied to each of the masses gives

$$T - m_1 g = m_1 a \quad \text{and} \quad m_2 g - T = m_2 a \tag{2}$$

where T is the tension in the string and a is the magnitude of the acceleration of either mass. Combining the two equations 2 leads to the following:

$$(m_2 - m_1)g = (m_1 + m_2)a \tag{3}$$

Essentially, equation 3 states that a force $(m_2 - m_1)g$ equal to the difference in the weight of the two masses acts on the sum of the masses $m_1 + m_2$ to produce an acceleration a of the system. In fact, there will be some frictional force f in the system that will tend to subtract from the applied force $(m_2 - m_1)g$. Including the frictional force but moving it to the other side of the equation gives

$$(m_2 - m_1)g = (m_1 + m_2)a + f \tag{4}$$

Consider the Atwood machine shown in Figure 9.2, where $m_2 > m_1$, the mass m_1 is initially on the floor, and m_2 is released from rest a distance x above the floor at $t = 0$. Successive positions of the two masses are shown in Figure 9.2 at later times until the final picture, which shows m_2 as it strikes the floor at some time t after its release.

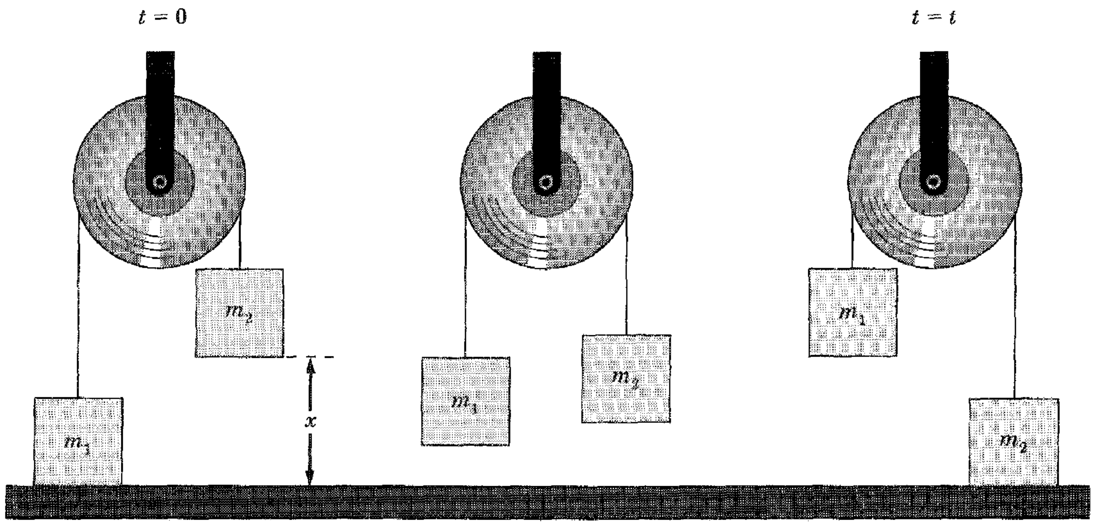

Figure 9.2 Atwood machine as mass m_2 falls a distance x in time t.

The relationship between the distance x, the acceleration a of the system, and the time t is given by

$$x = \frac{at^2}{2} \tag{5}$$

Solving equation 5 for a in terms of the measured quantities x and t gives

$$a = \frac{2x}{t^2} \tag{6}$$

This laboratory will measure the acceleration for the Atwood machine for several different values of the applied force $(m_2 - m_1)g$ using a fixed total mass $m_1 + m_2$.

EXPERIMENTAL PROCEDURE

1. Using the laboratory balance, place 1.000 kg of slotted masses (including a 0.050-kg-mass holder) on each pan of the balance. Include five 0.002-kg masses on one of the pans. If all the masses are accurate, the scale should be balanced. If, however, the scales are not exactly balanced, add 0.001-kg slotted masses as necessary to produce as perfect a balance as can be obtained to the nearest 0.001 kg. At most, one or two 0.001-kg masses should be needed. The purpose of this procedure is to produce two masses that are as nearly equal as possible. Record in the Data Table the sum of all mass on the balance as $m_1 + m_2$.
2. Place these two collections of slotted masses on their mass holders at each end of a string over the pulley. Place the group of masses that contain the collection of 0.0020-kg masses on the left side (m_1) of the pulley and the other masses on the right (m_2).
3. Transfer a 0.0020-kg mass from the left side (m_1) to the right side (m_2) while holding the system fixed. This produces a mass difference ($m_2 - m_1$) of 0.0040 kg.

4. While one partner holds mass m_1 on the floor, another partner should measure the distance x from the bottom of mass m_2 to the floor as shown in Figure 9.2. The height of the pulley above the floor should be chosen so that x is at least 1.000 m. Record this distance as x in the Data Table. *Use this same distance for all measurements.*

5. Release mass m_1 and simultaneously start the timer. Stop the timer when mass m_2 strikes the floor. Repeat this measurement four more times, for a total of five trials with a mass difference $m_2 - m_1$ of 0.0040 kg. Record all times in the Data Table.

6. Repeat steps 4 and 5 using mass differences $m_2 - m_1$ of 0.0080, 0.0120, 0.0160, and 0.0200 kg by transferring an additional 0.0020 kg each time. Make a total of five trials for each mass difference and record the measured times in the Data Table.

CALCULATIONS

1. Calculate the applied force $(m_2 - m_1)g$ for each of the mass differences and record the results in the Calculations Table. Use a value of 9.80 m/s^2 for g.

2. Calculate the mean time $\bar{t}$ and the standard error α_t for the five measurements of time at each of the mass differences $m_2 - m_1$. Record those values in the Calculations Table.

3. Using equation 6, calculate the acceleration a from the measured values of x and $\bar{t}$ for each value of applied force. Record these values of a in the Calculations Table.

4. According to equation 4, the applied force $(m_2 - m_1)g$ should be proportional to the acceleration a with the total mass $m_1 + m_2$ as the constant of proportionality and the frictional force f as the intercept. Perform a linear least squares fit to the data with the applied force as the ordinate and the acceleration as the abscissa. Calculate the slope of the fit and record it as $(m_1 + m_2)_{exp}$ in the Calculations Table. Calculate the intercept of the fit and record it as f in the Calculations Table.

5. Calculate the percentage error in the value of $(m_1 + m_2)_{exp}$ compared to the known value of $m_1 + m_2$ and record it in the Calculations Table.

GRAPHS

Make a graph of the data for the applied force versus the acceleration. Also show on the graph the straight line obtained by the least squares fit to the data.

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LABORATORY REPORT

Data Table

$m_1 + m_2 =$ ____________ kg $\quad x =$ ____________ m

$m_2 - m_1$ (kg)	t_1 (s)	t_2 (s)	t_3 (s)	t_4 (s)	t_5 (s)
0.0040					
0.0080					
0.0120					
0.0160					
0.0200					

Calculations Table

$(m_2 - m_1)\,g$ (N)	$\bar{t}$(s)	a_t (s)	a (m/s^2)

$(m_1 - m_2)_{exp} =$ ________ kg $\quad f =$ ________ N $\quad r =$ ________ $\quad$ % err = ________

SAMPLE CALCULATIONS

QUESTIONS

1. Do your data and graph of the applied force versus the acceleration show the linear relationship that is predicted by Newton's second law? Consider the value of r, the regression coefficient, and the table in Appendix I in your answer.

2. Is your experimental value for $m_1 + m_2$ obtained from the slope of applied force versus acceleration larger or smaller than the actual value of $m_1 + m_2$?

3. If friction is truly a constant for all applied forces it will not affect the slope of the data. If, however, the frictional force is a function of the applied force (f either increases or decreases with applied force), that would tend to change the slope of the data and thus the experimental mass. Which way would the force have to change (i.e., get larger with the applied force or get smaller with the applied force) in order to account for the results stated in question 2?

4. Express the frictional force as a percentage of the applied force, $(m_1 - m_2)g$, for each applied force. Based on these percentages, does friction appear to be in the right direction and of the approximate size to account for the difference in the measured mass?

5. It can be shown that if the pulley is assumed to be a uniform disk, the effect of its moment of inertia would be equivalent to adding a mass equal to one-half the pulley mass to $m_1 + m_2$. What value does the error in your results imply for the mass of the pulley?

6. Measure the mass of the pulley or obtain its value from your instructor. Based on this value and your answer to question 5, could the pulley mass account for a significant portion of your error in the experimental value of $m_1 + m_2$?

7. Calculate the tension T in the string for each of the five applied forces.