Name ______________________ Section ____________ Date ____________

Laboratory 46

Nuclear Counting Statistics

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. For nuclear counting experiments no true value of a given count is assumed. What quantity is assumed to have a true value?

2. What is the exact statistical distribution function that describes the statistics of nuclear counting experiments?

3. What statistical distribution function approximates nuclear counting statistics and is used because it deals with continuous variables? For what values of the true mean m is this distribution valid?

4. In a nuclear counting experiment, a single measurement of C counts is obtained. What is the approximate value for σ_{n-1} for the count C?

5. According to the normal distribution function for the case when a given count is repeated 30 times, approximately how many of the results should fall in the range $\overline{C} \pm \sigma_{n-1}$? How many should fall in the range $\overline{C} \pm 2\sigma_{n-1}$?

6. A single count of a radioactive nucleus is made, and the result is 927 counts. What is the approximate value of σ_{n-1}?

7. A set of 10 repeated measurements of the count from a given radioactive sample were taken. The results were 633, 666, 599, 651, 654, 690, 660, 659, 664, and 612. What is the mean count $\overline{C}$? What is the value of σ_{n-1}? What is the value of α? Which of the counts fall outside $\overline{C} \pm \sigma_{n-1}$? Is this approximately the number of cases expected?

8. For the data in question 7, is $\sqrt{\overline{C}}$ approximately equal to σ_{n-1}?

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Nuclear Counting Statistics

OBJECTIVES

Consider a sample of long-lived radioactive nuclei. If the number of nuclei that decay in some fixed time interval is measured several times, there will be some variation in the count obtained for the different trials. This will be true even if all experimental errors have been eliminated. This variation is caused by the random nature of the nuclear decay process itself. The average count $\overline{C}$ can be determined with high precision by taking a large number of trials of the count for the given time period. This laboratory will use a Geiger counter to make repeated measurements of the number of counts from a long-lived radioactive isotope to accomplish the following objectives:

1. Determination of the average count in a fixed time interval for fixed experimental conditions by 50 independent determinations of the count
2. Determination of the standard deviation from the mean and standard error of the individual measurements
3. Comparison of the observed distribution of the individual counts relative to the mean compared to what is predicted by the normal distribution
4. Demonstration that $\sqrt{C}$ is an approximation of the standard deviation from the mean for a single measurement resulting in C

EQUIPMENT LIST

1. Geiger counter (single unit containing Geiger tube, power supply, timer, and scaler)
2. Long-lived radioactive source (such as ^{137}Cs or ^{60}Co)

THEORY

If all other sources of error are removed from a nuclear counting experiment, there remains an uncertainty due to the random nature of the nuclear decay process. It is assumed that there exists some true mean value of the count that shall be designated as m. However, it is extremely important to realize that it is not assumed that there is a true value for any individual count C_i. Although m is assumed to exist, it can never be known exactly. Instead, one can approach knowledge of the true mean

m by a large number of observations. It can be shown that the best approximation to the true mean m is the mean $\overline{C}$, which is given by

$$\overline{C} = \frac{1}{n}\sum_{i=1}^{n} C_i \tag{1}$$

where C_i stands for the ith value of the count obtained in n trials. The standard deviation from the mean σ_{n-1} and the standard error α are defined in the usual manner as

$$\sigma_{n-1} = \sqrt{\sum_{i=1}^{n} \frac{1}{n-1}(\overline{C} - C_i)^2} \quad \text{and} \quad \alpha = \frac{\sigma_{n-1}}{\sqrt{n}} \tag{2}$$

These are no different from the assumptions that have been applied to essentially all of the measurements in this laboratory manual. The only difference is that in many of the cases in which these ideas have been applied, they are somewhat questionable because the random errors are not necessarily the determining factor. For nuclear counting experiments it is usually the case that the random errors are the limiting factor, and these concepts generally do apply strictly to such measurements.

The manner in which the measurements C_i are distributed around the mean $\overline{C}$ depends on the statistical distribution. The binomial distribution is the fundamental law for the statistics of all random events, including radioactive decay. Calculations are difficult with this distribution, and it is often approximated by another integral distribution called the "Poisson distribution." For cases of m greater than 20, both the binomial and the Poisson distribution can be approximated by the normal distribution. It has the advantage that it deals with continuous variables, and thus calculations are much easier with the normal distribution. The result is that for most nuclear counting problems of interest, the normal distribution predicts the same results for nuclear counting that has been assumed for measurements in general. These results are that approximately 68.3% of the measured values of C_i should fall within $\overline{C} \pm \sigma_{n-1}$, and approximately 95.5% of the measured values of C_i should fall within $\overline{C} \pm 2\sigma_{n-1}$.

There is one further statistical idea valid for nuclear counting experiments that is not true for measurements in general. For any given single measurement of the count C in a nuclear counting experiment, an approximation to the standard deviation from the mean σ_{n-1} is given by

$$\sigma_{n-1} \approx \sqrt{C} \tag{3}$$

For a series of repeated trials of a given count, the most accurate determination is given by $\overline{C} \pm \alpha$. If only a single measurement of the count is made, the most accurate statement that can be made is given by $C \pm \sqrt{C}$.

In this laboratory, a series of measurements of the same count will be made to determine the distribution of the measurements about the mean. In addition, the validity of equation 3 will be investigated.

EXPERIMENTAL PROCEDURE

1. Consult your instructor for the operating voltage of the Geiger counter. (Figure 46.1) It may be necessary to perform a quick set of measurements to determine the Geiger plateau. If so, consult Laboratory 45 for the procedure.
2. Set the Geiger counter to the proper operating voltage. Place a long-lived radioactive isotope on whichever counting shelf is necessary to produce between 500 and

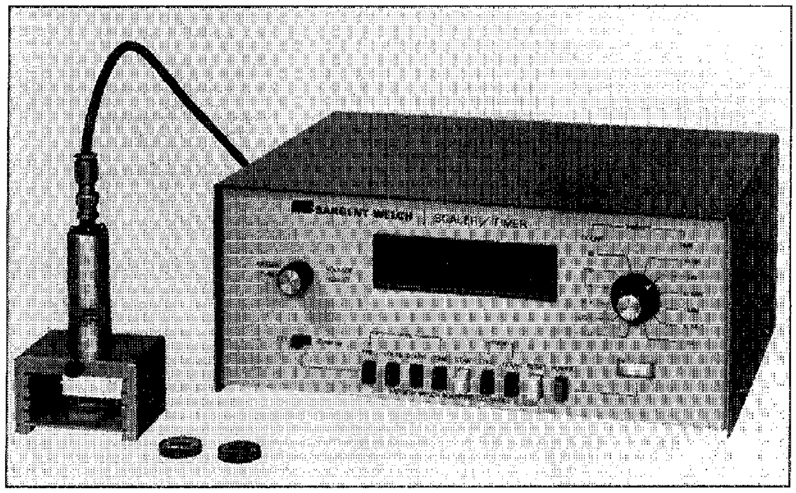

Figure 46.1 Geiger counter with timer-scaler and encapsulated radioactive sources. (Photo courtesy of Sargent-Welch Scientific Company)

700 counts in a 30-s counting period. For best results, the Geiger counter should have preset timing capabilities. If it does not and a laboratory timer is used, it would improve the timing precision if 60-s counting intervals are used. For whatever time is counted, between 500 and 700 counts should be recorded.

3. Repeat the count for a total of 50 trials. Make no changes whatsoever in the experimental arrangement for these 50 trials. Record each count in the Data Table. Do not make any background subtraction. Simply record the total count for each counting period.

CALCULATIONS

1. Calculate the mean count $\overline{C}$, the standard deviation from the mean σ_{n-1}, and the standard error α for the 50 trials of the count and record the results in the Calculations Table.
2. For each count C_i calculate $|C_i - \overline{C}|/\sigma_{n-1}$ and record the results in the Calculations Table.
3. Determine what percentage of the counts C_i are further from $\overline{C}$ than σ_{n-1} by counting the number of times a value of $|C_i - \overline{C}|/\sigma_{n-1} > 1$ occurs. Express this number divided by 50 as a percentage. Count the number of times that $|C_i - \overline{C}|/\sigma_{n-1} > 2$ occurs. Express this number divided by 50 as a percentage. Record these results in the Calculations Table.
4. Calculate $\sqrt{\overline{C}}$ and record its value in the Calculations Table.

GRAPHS

1. Construct a histogram of your data on linear graph paper. Consider the range of the data and arbitrarily divide the range into about 15 segments. For counts in the range used this should give intervals of 8 or 10 counts. An example of some data is displayed in this manner in Figure 46.2. The mean of the data is 659 with σ_{n-1} = 27, and an interval of 10 has been chosen.

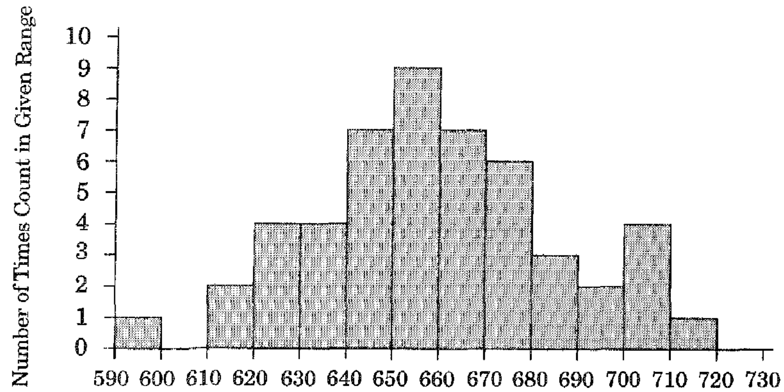

Figure 46.2 Histogram of 50 repeated counts with mean of 659.

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Laboratory 46

Nuclear Counting Statistics

LABORATORY REPORT

Data Table

i	C_i	i	C_i
1		26	
2		27	
3		28	
4		29	
5		30	
6		31	
7		32	
8		33	
9		34	
10		35	
11		36	
12		37	
13		38	
14		39	
15		40	
16		41	
17		42	
18		43	
19		44	
20		45	
21		46	
22		47	
23		48	
24		49	
25		50	

Calculations Table

i	$\lvert C_i - \bar{C} \rvert / \sigma_{n-1}$	i	$\lvert C_i - \bar{C} \rvert / \sigma_{n-1}$
1		26	
2		27	
3		28	
4		29	
5		30	
6		31	
7		32	
8		33	
9		34	
10		35	
11		36	
12		37	
13		38	
14		39	
15		40	
16		41	
17		42	
18		43	
19		44	
20		45	
21		46	
22		47	
23		48	
24		49	
25		50	

$\bar{C} =$	$\sigma_{n-1} =$
$\alpha =$	$\sqrt{\bar{C}} =$
% trials > σ_{n-1} from mean =	
% trials > $2\sigma_{n-1}$ from mean =	

SAMPLE CALCULATIONS

QUESTIONS

1. Consider the shape of the histogram of your data. Does it show the expected distribution relative to the mean of the data?

2. Compare the percentage of trials that have $|C_i - \overline{C}|/\sigma_{n-1} > 1$ with that predicted by the normal distribution. Compare the percentage of trials that have $|C_i - \overline{C}|/\sigma_{n-1} > 2$ with that predicted by the normal distribution.

3. What is the most accurate statement that you can make about the count from the sample based on the data that you have taken?

4. Calculate the percentage difference between $\sqrt{\overline{C}}$ and σ_{n-1}. Do the results confirm the expectations of equation 3?

Suppose that you were to perform another 50 trials of the count of the same sample under the exact same conditions as the first 50 trials. Answer questions 5 to 7 about what would be expected for the total 100 trials that you now have.

5. Would the mean $\overline{C}$ for the 100 trials be expected to be significantly different from the mean $\overline{C}$ for the first 50 trials?

6. Would the standard deviation from the mean σ_{n-1} for the 100 trials be expected to be significantly different from the standard deviation from the mean σ_{n-1} for the first 50 trials?

7. Would the standard error α for the 100 trials be expected to be significantly different from the standard error α for the first 50 trials?