Name ______________________ Section ____________ Date ____________

Laboratory 36

Alternating-Current *LR* Circuits

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. For a resistor in a series alternating-current circuit, the phase relationship between the current in the resistor and the voltage across the resistor is (a) the current leads the voltage by 90°, (b) the voltage leads the current by 90°, (c) the current is in phase with the voltage, or (d) the current is at some phase angle ϕ relative to the voltage (ϕ is dependent on the circuit parameters).
2. For an inductor in a series alternating-current circuit, the phase relationship between the current in the inductor and the voltage across the inductor is (a) the current leads the voltage by 90°, (b) the voltage leads the current by 90°, (c) the current is in phase with the voltage, or (d) the current is at some phase angle ϕ relative to the voltage (ϕ is dependent on the circuit parameters).
3. For a generator in a series alternating-current circuit, the phase relationship between the generator voltage and the current in the generator is (a) the current leads the voltage by 90°, (b) the voltage leads the current by 90°, (c) the current is in phase with the voltage, or (d) the current is at some phase angle ϕ relative to the voltage (ϕ is dependent on the circuit parameters).
4. If a generator has a maximum voltage of 5.00 V, what is the root-mean-square voltage of the generator? Show your work.

   V_{rms} = ______________________

5. A 2.50-mH inductor has an rms voltage of 15.0 V across it at a frequency $f = 200$ Hz. What is the rms current in the inductor? Show your work.

6. A pure inductor L and a pure resistor R are in series with a generator of voltage V. The voltage across the inductor is $V_L = 10.0$ V. The voltage across the resistor is 15.0 V. What is the voltage V of the generator? Show your work.

$V =$ ____________________

7. A 500-Ω resistor and a real inductor whose pure inductance is L and whose internal resistance is r are in series with a generator whose voltage is $V = 10.0$ V and whose angular frequency is $\omega = 1000$ rad/s. The voltage across the real inductor is measured to be 4.73 V, and the voltage across the 500-Ω resistor is measured to be 6.57 V. What is the value of L and r? (*Hint*: This is the measurement to be performed in this laboratory exercise. Use equation 7 to find ϕ, use equations 8 and 9 to find V_L and V_r, and then use equations 11 and 12 to find ωL and r. Finally find L from the known value of ω.) Show your work.

Laboratory 36

Alternating-Current LR Circuits

OBJECTIVES

If a sinusoidally varying source of emf with a frequency f is placed in series with a resistor and a pure inductor, the current I varies with time but is the same in each element of the circuit at any given instant. This current I will vary with the same frequency f as the generator, but it will be shifted in phase by an angle ϕ relative to the generator. The voltage across each of the circuit elements has its own characteristic phase relationship with the current. The resistor voltage V_R is in phase with the current I, the inductor voltage V_L leads the current by a phase angle of 90°, and the generator voltage leads the current by an angle ϕ, whose value is dependent on the circuit parameters. Measurements of the voltage across each element in a series circuit of an inductor, a resistor, and a sine-wave generator will be used to accomplish the following objectives:

1. Demonstration of the relative phase angle of the voltage across each component in the circuit
2. Determination of the phase angle of the generator current relative to the generator voltage
3. Demonstration that real inductors consist of both inductance and resistance, and that they can be represented by a pure inductor L in series with a pure resistance r
4. Determination of the value of L and r for an unknown inductor

EQUIPMENT LIST

1. Sine-wave generator (variable frequency, 5 V peak-to-peak amplitude)
2. Resistance box
3. A 100-mH inductor (resistance $\cong$ 350 Ω to serve as an unknown)
4. Alternating-current voltmeter (digital readout, capable of measuring high frequency)
5. Compass and protractor

THEORY

Consider first the two circuits shown in Figure 36.1 in which a sine-wave generator of frequency f is connected separately to resistor R and then to a pure inductance L. The generator is assumed to have a maximum voltage of V and will thus produce a maximum voltage of V across the resistor in circuit (a). It will also produce a maximum voltage of V across the inductor in circuit (b). The voltage across the resistor is related to the current by a relationship like that for direct current circuits which is

$$V_R = IR \tag{1}$$

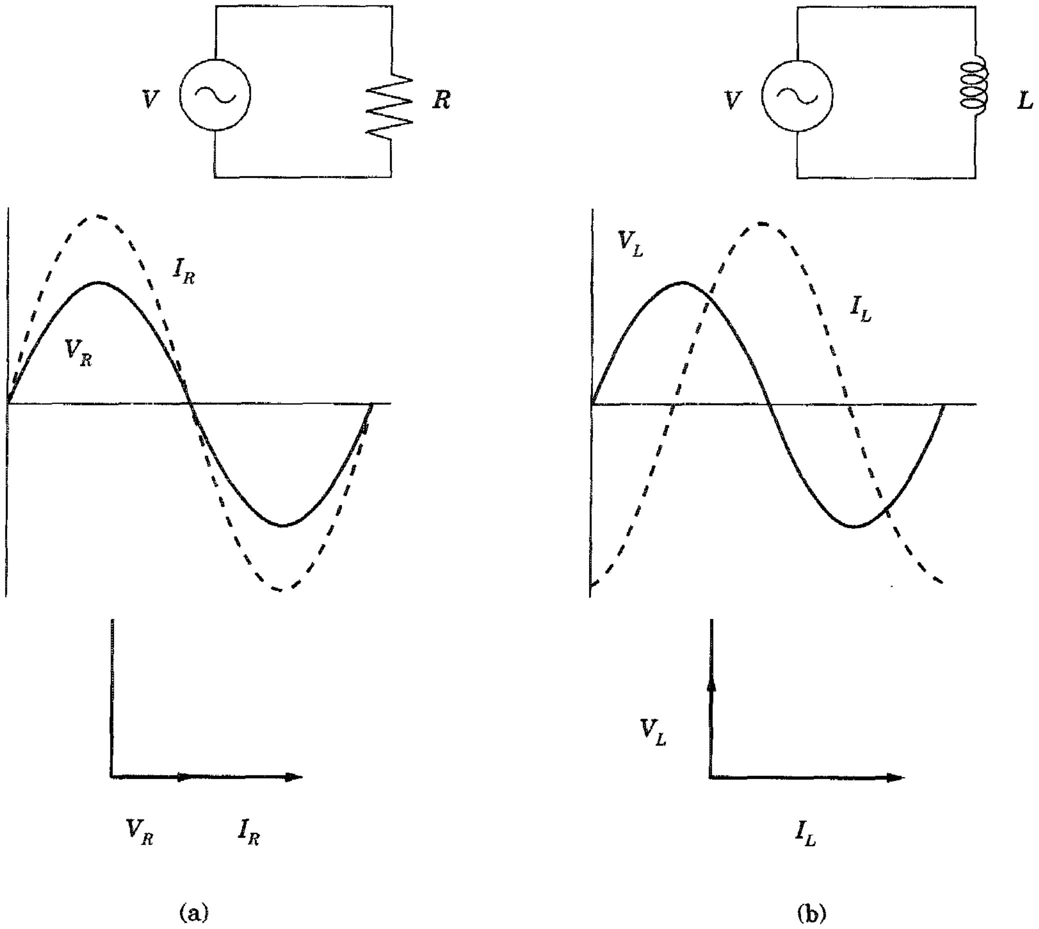

Figure 36.1 Generator and Resistor and Generator and Inductor. Phase relationships between the voltage and current and phasor diagrams of the phase relationships.

If L is the inductance (in units of H) and ω is the angular frequency of the generator ($\omega = 2\pi f$) in rad/s, then the following relationship exists between the voltage V and the current I

$$V_L = I\,\omega L \tag{2}$$

The quantity ωL is called the "inductive reactance," and it has units of Ω. For example, if a 1.50-mH inductor has a voltage of 10.0 V across it at a frequency of 100 Hz, the current is $I = V_L/\omega L = 10.0/2\pi(100)(1.50 \times 10^{-3}) = 10.6$ A.

When an alternating current or voltage is measured in the laboratory on a meter, the number read for the current or voltage must be some time-averaged value. Meters are normally calibrated in such a way that they respond to the root-mean-square value of the current or voltage. A root-mean-squared value of voltage can be designated as V_{rms}. The relationship between V_{rms} and V the maximum voltage is

$$V_{rms} = \frac{\sqrt{2}}{2} V = 0.707\ V \tag{3}$$

Note that equations 1 and 2 refer to either maximum values or rms values. If the voltage is expressed as a maximum value of voltage in those equations, then the current will be a maximum value. If the voltage is an rms value, then the current will be an rms value. In this laboratory the only quantities measured will be voltage, and all the measurements will be rms values. No subscripts will be put on the symbols, but it is understood that all measured values of voltage are rms values.

Also shown in Figure 36.1 below each circuit is a graph of the current and voltage across the element for one full period. The graph for the case of the resistor indicates that the resistor current I_R and the resistor voltage V_R are in phase. For the inductor the graph shows that the inductor current I_L and the inductor voltage V_L are 90° out of phase, with the voltage leading the current by 90°. The graph below also shows that the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value.

Shown at the bottom of Figure 36.1 is a diagram called a "phasor diagram," whose purpose is also to show the phase relationship. The phasors are vectors drawn with length proportional to the value of the represented quantity, and they are assumed to be rotating counterclockwise with the frequency of the generator. At any time, a projection of one of the rotating vectors on the y-axis is the instantaneous value of that quantity. Since the resistor current and voltage are in phase, they are shown in the same direction, and therefore their projection of the y-axis is always in phase. For the inductor the vector representing the inductor voltage is 90° ahead of the vector representing the current when they are both assumed to be rotating counterclockwise.

Consider now the circuit obtained by placing a pure inductance L having no resistance and a resistor R in series with a sine-wave generator of voltage V shown in Figure 36.2. For this circuit, the current I is the same at every instant of time in all three circuit elements. Also given in Figure 36.2 is a phasor diagram in which only the voltages are shown. The phasor representing the current (which is not shown) would be in the direction of the phasor labeled V_R because the current and the resistor voltage are in phase. Note that the inductor voltage V_L is 90° ahead of the resistor voltage V_R, and the generator voltage is angle ϕ ahead of V_R. This phasor diagram shows that the generator voltage V is the vector sum of V_R and V_L. In equation form the phasor diagram states

$$V = \sqrt{V^2{}_L + V^2{}_R} \tag{4}$$

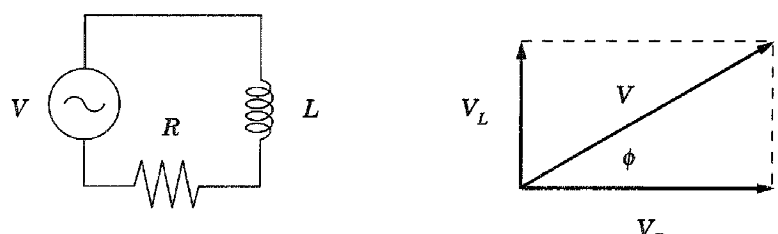

Figure 36.2 Series circuit of Resistor and Inductor and associated phasor diagram.

From an examination of the phasor diagram it is also clear that the phase angle ϕ is related to the voltages V_L and V_R, and thus to the resistance R and ωL through equations 1 and 2. The relationship is given by

$$\tan \phi = \frac{V_L}{V_R} = \frac{\omega L}{R} \tag{5}$$

Note that equation 5 is strictly valid only for a pure inductor that has no resistance, but it can be used as an approximation if the resistance of the inductor is negligible compared to the resistor R.

In fact, real inductors do have both an inductance L and an internal resistance r. Essentially, a real inductor can be represented by a pure inductance L in series with a pure resistance r. In Figure 36.3 a real inductor is shown in series with a resistor R and a generator of voltage V. The points A, B, and C in the figure represent the three points between which a voltmeter can be placed in the circuit in order to measure a voltage. The voltage between points A and B is the generator voltage V, and the voltage between A and C is the resistor voltage V_R. Between the points B and C is the combined voltage across the inductance L and the internal resistance r. This voltage will be referred to as V_{ind}. There is, of course, some voltage V_L across L alone, and some voltage V_r across r alone. However, there can be no direct measurement of V_L or V_r. The only quantity that can be measured is V_{ind}, which is the vector sum of V_L and V_r.

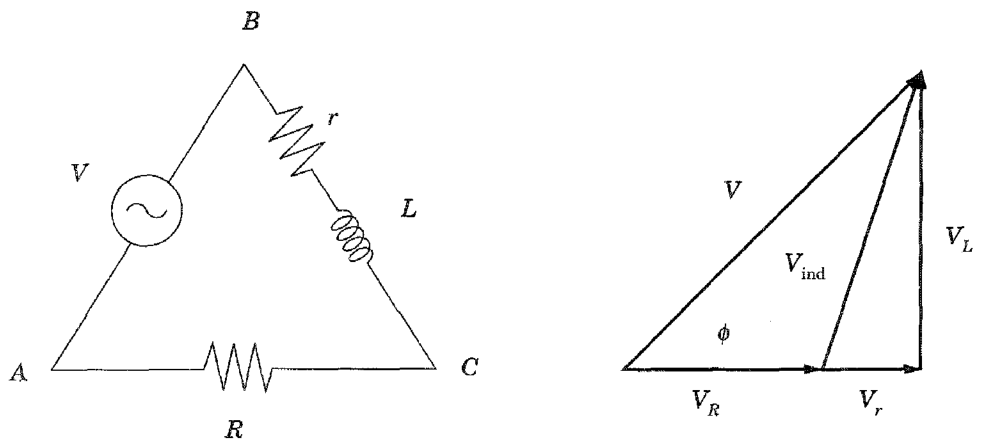

Figure 36.3 Series circuit of real inductor with internal resistance r, a resistor R, and a generator of voltage V. Also shown is the associated phasor diagram of the voltages.

A phasor diagram for the circuit is also shown in Figure 36.3. Applying the law of cosines to the triangle formed by V, V_R, and V_{ind} leads to

$$V^2_{ind} = V^2 + V^2_R - 2VV_R \cos \phi \qquad \textbf{(6)}$$

Solving equation 6 for $\cos\phi$ leads to the following:

$$\cos \phi = \frac{V^2 + V^2_R - V^2_{ind}}{2VV_R} \qquad \textbf{(7)}$$

Equation 7 states that the angle ϕ can be determined by a measurement of V, V_R, and V_{ind}.

An examination of the phasor diagram in Figure 36.3 shows that the unknown voltages V_L and V_r can be determined from V, V_R, and ϕ by the following:

$$V_L = V \sin\phi \qquad \textbf{(8)}$$

$$V_r = V \cos\phi - V_R \qquad \textbf{(9)}$$

The current I is the same in all the elements of the circuit, and it can be related to the voltage across each element by the following equations:

$$V_L = I\omega L \qquad V_R = IR \qquad V_r = Ir \qquad \textbf{(10)}$$

Once V_L and V_r have been determined from equations 8 and 9, the equations in 10 can be used to solve for ωL and r by eliminating I to get

$$\omega L = R\frac{V_L}{V_R} \tag{11}$$

$$r = R\frac{V_r}{V_R} \tag{12}$$

EXPERIMENTAL PROCEDURE

1. Connect the inductor in series with the sine-wave generator and a resistance box to form a circuit like that of Figure 36.3. Set the generator to maximum voltage and a frequency of 800 Hz. Set the resistance box to a value of 400 Ω and record that value as R in the Data Table.
2. Using the AC voltage scale on the voltmeter, very carefully measure the generator voltage V, the inductor voltage V_{ind}, and the voltage across the resistor V_R. Record these values in the Data Table.
3. Repeat steps 1 and 2 for values of R of 600, 800, and 1000 Ω. Do not assume that the generator voltage stays the same. Even though the voltage setting is left at the maximum setting, the generator output might change slightly in response to the changes in R. Therefore, be sure to measure all three voltages for each value of R.
4. Make careful note of the particular inductor you used. You may need to identify it and use it again in other laboratory exercises involving an inductor in an alternating current circuit. The values that are determined for L and r should be noted and saved along with the means of identification of the inductor.

CALCULATIONS

1. From the known value of the frequency f, calculate and record in the Calculations Table the value of the angular frequency ω ($\omega = 2\pi f$).
2. Using equation 7, calculate the value of $\cos\phi$ and then of ϕ for each case and record the values in the Calculations Table.
3. Using equations 8 and 9, calculate V_L and V_r for each case and record the values in the Calculations Table.
4. Using equations 11 and 12 calculate ωL and r for each of the four cases and record the values in the Calculations Table.
5. Calculate the four values for L from the four values of ωL and the known value of ω and record them in the Calculations Table.
6. Calculate the mean and standard errors for the four values of r and the four values of L and record them in the Calculations Table as $\bar{r}$, $\bar{L}$, α_r, and α_L.

GRAPHS

Construct to scale a phasor diagram like the one shown in Figure 36.4 for each of the four cases. Use one sheet of graph paper and make four separate diagrams on the one sheet of paper. Choose some scale (for example, 1.00 V = 1.00 cm) so that the diagrams are as large as possible, and that each one fits on one fourth of the sheet of paper. First, construct a vector along the x-axis whose length is scaled to the magnitude of V_R as shown in Figure 36.4. Using a compass construct an arc from the end of V_R whose radius is the length of the scaled value of V_{ind}. Finally, construct an arc from the beginning of V_R whose radius is the length of the scaled value of V. The intersection of the two arcs is the intersection of V_{ind} and V, and those two vectors can then be drawn in their proper direction as shown in part (b) of the figure. Finally V_L and V_r can be constructed as shown in part (c) of the figure by dropping a perpendicular from the intersection of the arcs to the x-axis and extending a vector from the end of V_R.

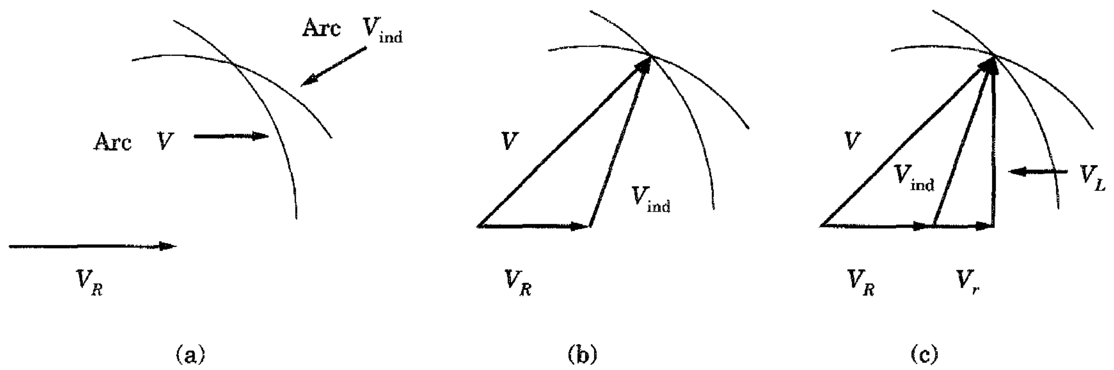

Figure 36.4 Phasor diagram construction.

Name ______________________ Section ____________ Date ____________

Laboratory 36

Alternating-Current LR Circuit

LABORATORY REPORT

Data Table

$\omega =$ rad/s				
R (Ω)				
V (V)				
V_{ind} (V)				
V_R (V)				

Calculations Table

$\cos\phi$				
ϕ (degrees)				
V_L (V)				
V_r (V)				
ωL (Ω)				
r (Ω)				
L (H)				
$\bar{r} =$ Ω	$\alpha_r =$ Ω	$\bar{L} =$ H	$\alpha_L =$ H	

SAMPLE CALCULATIONS

QUESTIONS

1. Comment on the precision of your measurement of L and r. State the evidence for your comments.

2. Examine the phasor diagrams that you have constructed. Using a protractor measure the angle ϕ of the constructed triangle of V, V_R, and V_{ind}. Compare it with the calculated value of ϕ for each of the phasor diagrams. Calculate the percentage error in the value of ϕ from the diagram compared to the calculated value.

3. If your inductor was used in a series circuit with a resistance of $R = 10{,}000\ \Omega$ and a generator of $\omega = 100{,}000$ rad/s, what would be the phase angle ϕ? (*Hint:* The resistance of the inductor would be negligible.)

4. Consider the circuit that you measured with $R = 600\ \Omega$. Calculate the value of the current from each of the three equations 10 and compare their agreement.