Name ______________________ Section __________ Date __________

Laboratory 33

The *RC* Time Constant

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. In a circuit such as the one in Figure 33.1 with the capacitor initially uncharged, the switch S is thrown to position A at $t = 0$. The charge on the capacitor (a) is initially zero and finally $C\mathcal{E}$, (b) is constant at a value of $C\mathcal{E}$, (c) is initially $C\mathcal{E}$ and finally zero, or (d) is always less than $\mathcal{E}/R$.

2. In a circuit such as the one in Figure 33.1 with the capacitor initially uncharged, the switch S is thrown to position A at $t = 0$. The current in the circuit is (a) initially zero and finally $\mathcal{E}/R$, (b) constant at a value of $\mathcal{E}/R$, (c) is equal to $C\mathcal{E}$, or (d) initially $\mathcal{E}/R$ and finally zero.

3. In a circuit such as the one in Figure 33.2, the switch S is first closed to charge the capacitor, and then it is opened at $t = 0$. The expression $V = \mathcal{E}\, e^{-t/RC}$ gives the value of (a) the voltage on the capacitor but not the voltmeter, (b) the voltage on the voltmeter but not the capacitor, (c) both the voltage on the capacitor and the voltage on the voltmeter, which are the same, or (d) the charge on the capacitor.

4. For a circuit such as the one in Figure 33.1, what are the equations for the charge Q and the current I as functions of time when the capacitor is charging?

   $Q =$ ______________ $I =$ ______________

5. For a circuit such as the one in Figure 33.1, what are the equations for the charge Q and the current I as functions of time when the capacitor is discharging?

   $Q =$ ______________ $I =$ ______________

6. If a 5.00-μF capacitor and a 3.50-MΩ resistor form a series RC circuit, what is the RC time constant? (Give proper units for RC.)

   $RC =$ ______________

7. Assume that a 10.0 μF capacitor, a battery of emf $\mathcal{E}$ = 12.0 V, and a voltmeter of 10.0 MΩ input impedance are used in a circuit such as that in Figure 33.2. The switch S is first closed, and then the switch is opened. What is the reading on the voltmeter 35.0 s after the switch is opened? Show your work.

V = ________________ V

8. Assume that a circuit is constructed such as the one shown in Figure 33.3 with a capacitor of 5.00 μF, a battery of 24.0-V, a voltmeter of input impedance 12.0 MΩ, and a resistor R_U = 10.0 MΩ. If the switch is first closed and then opened, what is the voltmeter reading 25.0 s after the switch is opened? Show your work.

V = ________________ V

9. In the measurement of the voltage as a function of time performed in this laboratory, the voltage is measured at fixed time intervals. (a) true (b) false

Laboratory 33
The RC Time Constant

OBJECTIVES

When a direct-current source of emf is suddenly placed in series with a capacitor and a resistor, there is current in the circuit for whatever time it takes to fully charge the capacitor. In a similar manner, there is a definite time needed to discharge a capacitor that has previously been charged. There is a characteristic time associated with either of these processes, called the "*RC* time constant," whose value depends on the value of the resistance R and the capacitance C. In this laboratory, series combinations of a power supply, a capacitor, and resistors will be used to accomplish the following objectives:

1. Demonstration of the finite time needed to discharge a capacitor
2. Measurement of the voltage across a resistor as a function of time
3. Determination of the *RC* time constant of two series *RC* circuits
4. Determination of the value of an unknown capacitor from measurements made on a series *RC* circuit using a voltmeter as the resistance
5. Determination of the value of an unknown resistor from measurements made on a second *RC* circuit with an unknown resistance in parallel with the voltmeter

EQUIPMENT LIST

1. Voltmeter (at least 10-MΩ input impedance, preferably digital readout)
2. Direct-current power supply (20 V)
3. Laboratory timer
4. High-quality capacitor (5–10 μF to serve as an unknown)
5. Resistor (approximately 10 MΩ to serve as an unknown)
6. Single-pole, double-throw switch
7. Assorted connecting leads

THEORY

Consider the circuit shown in Figure 33.1 consisting of a capacitor C, a resistor R, a source of emf $\mathcal{E}$, and a switch S. If the switch S is thrown to point A at time $t = 0$ when the capacitor is initially uncharged, charge begins to flow in the series circuit consisting of $\mathcal{E}$, R, and C and flows until the capacitor is fully charged. It can be shown that the current I starts at an initial value of $\mathcal{E}/R$ and decreases exponentially with time. The charge Q on the capacitor, on the other hand, begins at zero and increases exponentially with time until it becomes equal to $C\mathcal{E}$. The equations that describe those events are

$$Q = C\mathcal{E}(1 - e^{-t/RC}) \quad \text{and} \quad I = \mathcal{E}/R\, e^{-t/RC} \tag{1}$$

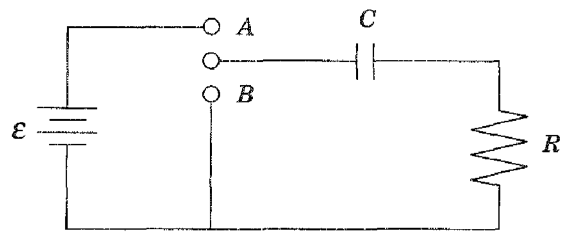

Figure 33.1 Simple Series *RC* Circuit.

The quantity RC is called the "time constant of the circuit," and it has units of seconds if R is expressed in ohms and C is expressed in farads. After a period of time that is long compared to the time constant RC, the term $e^{-t/RC}$ becomes negligibly small. When this is true the equations above predict that the charge Q is equal to $C\mathcal{E}$, and the current in the circuit is zero.

If switch S is now thrown to position B, which effectively takes $\mathcal{E}$ out of the circuit, the capacitor discharges through the resistor. Therefore, the charge on the capacitor and the current in the circuit both decay exponentially while the capacitor is discharging. The equations that describe this discharging process are

$$Q = C\mathcal{E}\, e^{-t/RC} \quad \text{and} \quad I = \mathcal{E}/R\, e^{-t/RC} \tag{2}$$

The equation for the current could be written with a negative sign because the current in the discharging case will be in the opposite direction from the current in the charging case. The magnitude of the current is the same in both cases. Although the above discussion has included both the case of charging and discharging a capacitor, this laboratory will only investigate the process of discharging a capacitor.

Consider the circuit shown in Figure 33.2 consisting of a power supply of emf $\mathcal{E}$, a capacitor C, a switch S, and a voltmeter whose input impedance is R. If initially the switch S is closed, the capacitor is charged almost immediately to $\mathcal{E}$, the voltage of the power supply. When the switch is opened the capacitor discharges through the resistance of the meter R with a time constant given by RC. With the switch open the only elements in the circuit are the capacitor C and the voltmeter resistance R; thus the voltage across the capacitor is equal to the voltage across the voltmeter. The voltage across the capacitor is given by Q/C, and the voltage across the voltmeter is given by IR. Solving 2 for those quantities leads in both cases to

$$V = \mathcal{E}e^{-t/RC} \tag{3}$$

Equation 3 stands either for the voltage across the voltmeter or the voltage across the capacitor as a function of time. Dividing both sides of equation 3 by $\mathcal{E}$ and taking the reciprocal of both sides of the equation leads to

$$\mathcal{E}/V = e^{-t/RC} \tag{4}$$

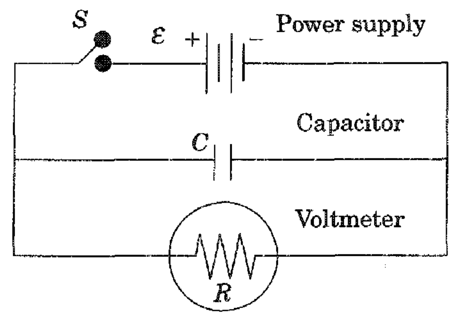

Figure 33.2 An RC circuit using a voltmeter as the resistance.

Taking the natural logarithm of both sides of equation 4 leads to the following:

$$\ln(\mathcal{E}/V) = (1/RC)\, t \tag{5}$$

Equation 5 states that there is a linear relationship between the quantity $\ln(\mathcal{E}/V)$ and the time t with the quantity $(1/RC)$ as the constant of proportionality. Therefore, if the voltage across the capacitor is determined as a function of time, a graph of $\ln(\mathcal{E}/V)$ versus t will give a straight line whose slope is $(1/RC)$. Thus, RC can be determined, and if the voltmeter resistance R is known, then C can be determined.

If an unknown resistor is placed in parallel with the voltmeter, it produces a circuit like that shown in Figure 33.3. The capacitor can again be charged and then discharged, but now the time constant will be equal to R_tC, where R_t is the total resistance, which is the parallel combination of R and R_U. If the relationship between R, R_U, and R_t is solved for R_U the result is

$$R_U = \frac{R\,R_t}{R - R_t} \tag{6}$$

Therefore, a measurement of the capacitor voltage as a function of time will produce a dependence like that given by equation 5 except that the slope of the straight line will be $(1/R_tC)$. Thus, if C is known and R_tC is found from the slope, then R_t can be determined. Using equation 6, R_U can be found from R and the value just determined for R_t.

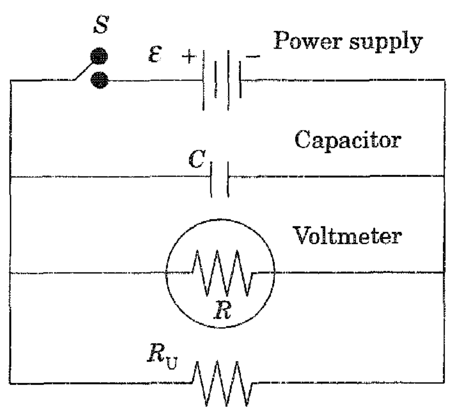

Figure 33.3 *RC* circuit using voltmeter and R_U in parallel as resistance.

EXPERIMENTAL PROCEDURE—UNKNOWN CAPACITANCE

1. Construct a circuit such as the one in Figure 33.2 using the capacitor supplied, the voltmeter, and the power supply. Have the circuit approved by your instructor before turning on any power. Obtain from your instructor the value of the input impedance of the voltmeter and record it in Data Table 1 as R.
2. Close the switch, and while reading its voltage on the voltmeter, adjust the power supply emf $\mathcal{E}$ to the value chosen by your instructor. Record the value of $\mathcal{E}$ in Data Table 1.
3. Open the switch and simultaneously start the timer.
4. The voltmeter reading will fall as the capacitor discharges. Let the timer run continuously, and for eight predetermined values of the voltage, record the time t on the timer when the voltmeter reads these voltages. A convenient choice for voltages at which to measure the time would be increments of 10%. For example, if $\mathcal{E}$ = 20.0 V, record the time when the voltage is 18.0, 16.0, 14.0, etc. Record the values of the voltage at which the time is to be read in Data Table 1 as V.
5. Record the values of t at which each value of V occurs in Data Table 1 under Trial 1.
6. Repeat steps 2 through 4 two more times, recording the values of t under Trials 2 and 3 in Data Table 1.

EXPERIMENTAL PROCEDURE—UNKNOWN RESISTANCE

1. Construct a circuit such as the one in Figure 33.3 using the same capacitor used in the last circuit and the unknown resistor supplied. Close the switch and adjust the power supply voltage to the same value used in the last procedure.
2. Repeat steps 2 through 6 of the procedure above, but record all values in the appropriate places in Data Table 2.

CALCULATIONS—UNKNOWN CAPACITANCE

1. Calculate the values of $\ln(\mathcal{E}/V)$ and record them in Calculations Table 1.
2. Calculate the mean $\bar{t}$ and the standard error α_t for the three trials of the time t at each voltage and record them in Calculations Table 1.
3. Perform a linear least squares fit of the data with $\ln(\mathcal{E}/V)$ as the ordinate and $\bar{t}$ as the abscissa.
4. The value of the slope is equal to $1/RC$. Record the value of the slope in Calculations Table 1. (The units of $1/RC$ are s^{-1}.)
5. Calculate RC as the reciprocal of the slope. Record the value of RC in Calculations Table 1. (The units of RC are s.)
6. Using the value of RC and the value of R, calculate the value of the unknown capacitor C and record it in Calculations Table 1.

CALCULATIONS—UNKNOWN RESISTANCE

1. Calculate the values of $\ln(\mathcal{E}/V)$ and record the values in Calculations Table 2.
2. Calculate the mean $\bar{t}$ and the standard error α_t for the three trials of the time t at each voltage and record them in Calculations Table 2.
3. Perform a linear least squares fit to the data with $\ln(\mathcal{E}/V)$ as the ordinate and $\bar{t}$ as the abscissa.
4. The value of the slope of this fit is equal to $1/R_tC$. Record the value of the slope in Calculations Table 2. (The units of $1/R_tC$ are s^{-1}.)
5. Calculate the value of R_tC as the reciprocal of the slope. Record the value of R_tC in Calculations Table 2. (The units of R_tC are s.)
6. Using the value of the capacitance C determined in the first procedure and the value of R_tC, calculate the value of R_t and record it in Calculations Table 2.
7. Using equation 6, calculate the value of the unknown resistance R_U from the values of R_t and R. Record the value of R_U in Calculations Table 2.

GRAPHS

1. For the data of Part 1, graph the quantity $\ln(\mathcal{E}/V)$ as the ordinate versus $\bar{t}$ as the abscissa. Use the values of the standard errors as error bars. Also, show on the graph the straight line obtained from the linear least squares fit to the data.
2. For the data of Part 2, graph the quantity $\ln(\mathcal{E}/V)$ as the ordinate versus $\bar{t}$ as the abscissa. Use the values of the standard errors as error bars. Also, show on the graph the straight line obtained from the linear least squares fit to the data.

Name ______________________ Section ____________ Date ____________

Laboratory 33

The *RC* Time Constant

LABORATORY REPORT

Data Table 1

V (V)	t_1 (s)	t_2 (s)	t_3 (s)
$\mathcal{E}$ =		V	
R =		Ω	

Calculations Table 1

ln ($\mathcal{E}$/V)	$\bar{t}$ (s)	α_t (s)
Regression coeff. =		
Intercept =		
Slope =		s^{-1}
RC =		s
C =		F

SAMPLE CALCULATIONS

Data Table 2

V (V)	t_1 (s)	t_2 (s)	t_3 (s)
$\mathcal{E}$ =		V	
R =		Ω	

Calculations Table 2

ln ($\mathcal{E}$/V)	$\bar{t}$ (s)	α_t (s)
Regression coeff. =		
Intercept =		
Slope =		s^{-1}
R_tC =		s
R_t =		Ω
R_U =		Ω

SAMPLE CALCULATIONS

QUESTIONS

1. Evaluate the linearity of each of the graphs. Do they confirm the linear dependence between the two variables that is predicted by the theory?

2. Ask your instructor for the values of the unknown capacitor and resistor. Calculate the percentage error of your measurement compared to the values provided. On this basis, evaluate the accuracy of your measurement of the capacitance and resistance.

3. Show that RC has units of seconds if R is in Ω and C is in F.

4. A 5.60-μF capacitor and a 4.57-MΩ resistor form a series RC circuit. If the capacitor is initially charged to 25.0 V, how long does it take for the voltage on the capacitor to reach 10.0 V? Show your work.