Name ____________________ Section __________ Date __________

Laboratory 20

Simple Harmonic Motion—Mass on a Spring

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. Describe in words and give an equation for the kind of force that produces simple harmonic motion.

2. Other than the type of force that produces it, what characterizes simple harmonic motion?

3. A spring has a spring constant $k = 8.75$ N/m. If the spring is displaced 0.150 m from its equilibrium position, what is the force that the spring exerts?

4. A spring of constant $k = 11.75$ N/m is hung vertically. A 0.500-kg mass is suspended from the spring. What is the displacement of the end of the spring due to the weight of the 0.500-kg mass?

5. A spring with a mass on the end of it hangs in equilibrium a distance of 0.4200 m above the floor. The mass is pulled down a distance 0.0600 m below the original position, released, and allowed to oscillate. How high above the floor is the mass at the highest point in its oscillation?

6. A massless spring has a spring constant of $k = 7.85$ N/m. A mass $m = 0.425$ kg is placed on the spring, and it is allowed to oscillate. What is the period T of oscillation?

7. Assume everything is the same as in question 6 except that the spring has a mass $m_s = 0.200$ kg. What is the period T of the system?

8. A massless spring of $k = 6.45$ N/m has a mass $m = 0.300$ kg on the end of the spring. The mass is pulled down 0.0500 m and released. What is the period T of the oscillation? What is the period T if the mass is pulled down 0.1000 m and released?

Simple Harmonic Motion—Mass on a Spring

OBJECTIVES

Simple harmonic motion is oscillatory motion that can be described by a single sine or cosine function. An object undergoes simple harmonic motion when it is subject to a force proportional to its displacement from an equilibrium position. In equation form $F = -kx$ describes such a force. A mass on the end of a spring is subject to a force that can be expressed by the above equation. In this laboratory, measurements of the motion of a mass on the end of a spring will be used to accomplish the following objectives:

1. Direct determination of the spring constant k of a spring by measuring the elongation of the spring for specific applied forces
2. Indirect determination of the spring constant k from measurements of the variation of the period T of oscillation for different values of mass on the end of the spring
3. Comparison of the two values of the spring constant k
4. Demonstration that the period T of oscillation of a mass on a spring is independent of the amplitude of the motion

EQUIPMENT LIST

1. Spring and masking tape
2. Table clamps, right angle clamps, and rods
3. Laboratory balance and calibrated hooked masses
4. Laboratory timer
5. Meter stick

THEORY

An object that experiences a restoring force proportional to its displacement from an equilibrium position is said to obey Hooke's law. In equation form this relationship can be expressed as

$$F = -kx \tag{1}$$

where k is a constant whose dimensions are N/m. The minus sign indicates that the force is in the opposite direction of the displacement. If a spring is the object exerting such a force, the constant k is called the "spring constant."

An object subject to a force described by equation 1 will undergo a type of oscillatory motion that is called "simple harmonic motion." The name comes from the fact that such motion can be described by a single sine or cosine function of time. If the object is displaced from its equilibrium position by some value A and then released, the object will oscillate back and forth about the equilibrium position. The values of its displacement x from the equilibrium position will range between $x = A$ and $x = -A$. The quantity A is called the "amplitude of the motion." For the initial conditions described above, the displacement x as a function of time t is given by

$$x = A \cos\left(\frac{2\pi t}{T}\right) \tag{2}$$

where T is the period of the motion. The period T is equal to the time for one complete oscillation from the maximum displacement on one side of equilibrium ($+A$), to the maximum displacement on the other side of equilibrium ($-A$), and back to the original position ($+A$).

Consider a mass m placed on the end of a spring hanging vertically as shown in Figure 20.1. The original equilibrium position of the lower end of the spring is shown in Figure 20.1(a), and the position of the lower end of the spring when the mass is applied is shown in Figure 20.1(b). For purposes of determining the oscillatory motion, the position shown in Figure 20.1(b) can be considered as the new equilibrium position, and displacements can be measured from that point. In Figure 20.1(c) the mass is shown pulled down to a displacement A from this equilibrium position. When released, the mass will oscillate with amplitude A.

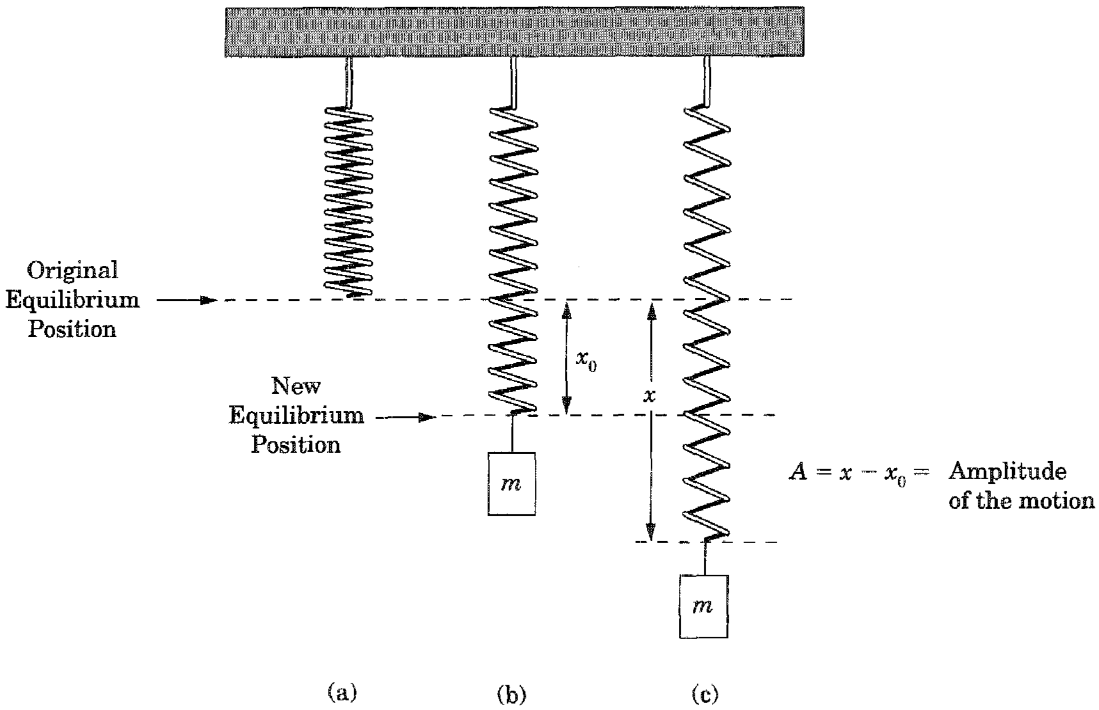

Figure 20.1 New equilibrium position with mass m placed on a spring.

The period of oscillation of the spring is independent of the amplitude A. It depends only on the spring constant k and the mass m. The period T is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{3}$$

Equation 3 is strictly true only if the spring is massless. For real springs with finite mass, a portion of the spring mass must be included along with the mass m. If the mass per unit length of the spring is constant, it can be shown that one third of the spring mass m_s must be included in equation 3 along with m. This gives as the equation for a spring of finite mass m_s

$$T = 2\pi \sqrt{\frac{m + (m_s/3)}{k}} \tag{4}$$

EXPERIMENTAL PROCEDURE—SPRING CONSTANT

1. Attach the table clamp to the edge of the laboratory table and screw a threaded rod into the clamp vertically as shown in Figure 20.2. Place a right-angle clamp on the vertical rod and extend a horizontal rod from the right-angle clamp. Hang the spring on the horizontal rod and attach it to the horizontal rod with a piece of tape. Screw a threaded vertical rod into a support stand that rests on the floor. Place a right-angle clamp on the vertical rod and place a meter stick in the clamp so that the meter stick stands vertically. Adjust the height of the clamp on the vertical rod until the zero mark of the meter stick is aligned with the bottom of the hanging spring as shown in Figure 20.2.

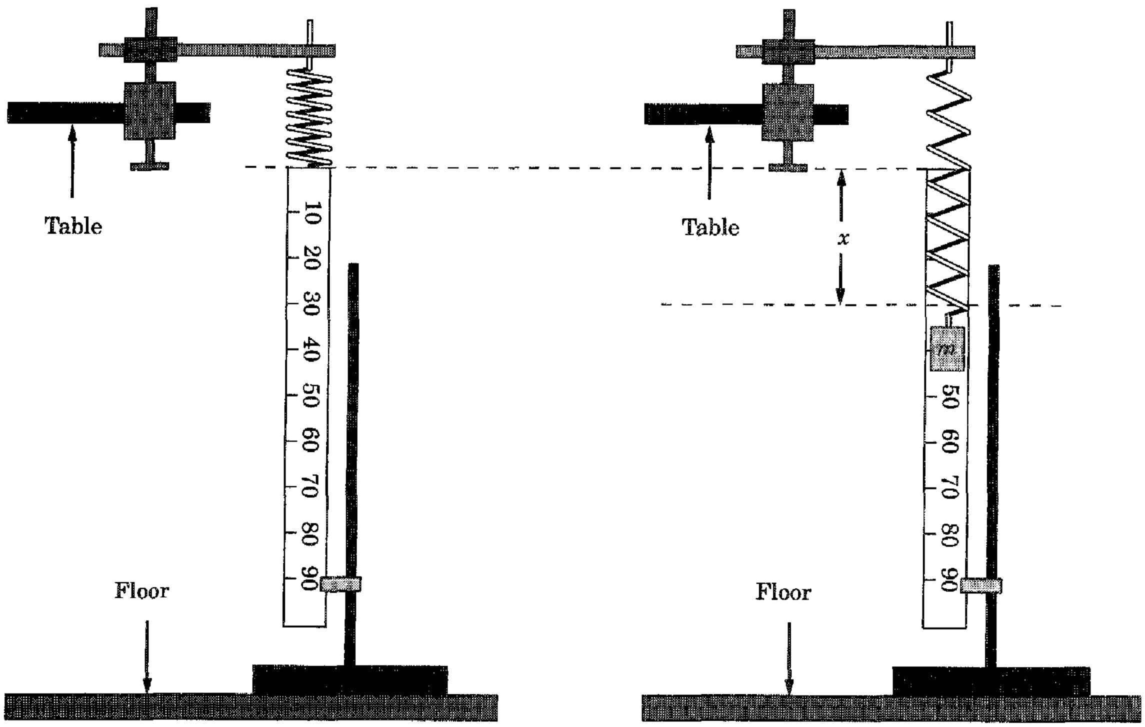

Figure 20.2 Arrangement to measure displacement of spring caused by mass m.

2. Place a hooked mass m of 0.1000 kg on the end of the spring. Slowly lower the mass m until it hangs at rest in equilibrium when released. Carefully read the position of the lower end of the spring on the meter-stick scale. Record the value of the mass m and the value of the displacement x in Data Table 1.
3. Repeat step 2, placing in succession 0.2000, 0.3000, 0.4000, and 0.5000 kg on the spring and measuring the displacement x of the spring. Record all values of m and x in Data Table 1.

CALCULATIONS—SPRING CONSTANT

1. Calculate the force mg for each mass and record the values in Calculations Table 1. Use the value of 9.800 m/s^2 for g. (It makes sense here to assume another significant figure in g in order not to be limited by that quantity for this data. Note that this is not the correct value to this number of significant figures but it makes no difference for our purposes.)
2. Perform a linear least squares fit to the data with mg as the ordinate and x as the abscissa. Calculate the slope of the fit and record it in Calculations Table 1 as the spring constant k.
3. Calculate the correlation coefficient for the linear least squares fit and record it in Calculations Table 1.

EXPERIMENTAL PROCEDURE—AMPLITUDE VARIATION

1. The dependence of the period T on the amplitude A will be done for a fixed mass. Place a hooked mass m of 0.2000 kg on the end of the spring. Slowly lower the mass until it hangs at rest when released. Note this position of the lower end of the spring.
2. Displace the mass downward 0.0200 m (i.e., so that $x - x_0 = A = 0.0200$ m as shown in Figure 20.1), release the mass, and let it oscillate. Measure the time for 10 complete periods and record it in Data Table 2 as Δt. Repeat the procedure two more times, for a total of three trials at this amplitude ($A = 0.0200$ m).
3. Repeat step 2 above for values of A equal to 0.0400, 0.0600, 0.0800, and 0.1000 m. Make three trials for each amplitude and measure the time for 10 periods for each trial. Record all results in Data Table 2.

CALCULATIONS—AMPLITUDE VARIATION

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t for the three trials for each amplitude. Record the results in Calculations Table 2.
2. Calculate the period T from $T = \overline{\Delta t}/10$. Record the results in Calculations Table 2.

EXPERIMENTAL PROCEDURE—MASS VARIATION

1. The data just taken for the period as a function of amplitude should have shown that the period is independent of the amplitude. Therefore, the dependence of the period T on the mass m can be done for any amplitude. Place a hooked mass of 0.0500 kg on the spring and let it hang at rest. Displace the mass slightly below the equilibrium, release it, and let the system oscillate. Measure the time for 10 periods of the motion and record that time in Data Table 3 as Δt. Repeat the procedure two more times, for a total of three trials with this mass.
2. Repeat the procedure of step 1 for values of the mass m equal to 0.1000, 0.2000, 0.3000, 0.4000, and 0.5000 kg. Perform three trials of the time for 10 periods for each mass and record the results in Data Table 3.
3. Using a laboratory balance, determine the mass of the spring m_s and record it in Data Table 3.

CALCULATIONS—MASS VARIATION

1. Calculate the mean $\overline{\Delta t}$ and standard error α_t for the three trials for each mass. Record the results in Calculations Table 3.
2. Calculate the period T from $T = \overline{\Delta t}/10$. Record the results in Calculations Table 3.
3. Calculate the quantity $\sqrt{m + (m_s/3)}$ for each of the values of m. Record the results in Calculations Table 3.
4. According to equation 4, the period T should be proportional to $\sqrt{m + (m_s/3)}$ with $2\pi/\sqrt{k}$ with T as the ordinate and $\sqrt{m + (m_s/3)}$ as the abscissa. Determine the slope of this fit and equate it to $2\pi/\sqrt{k}$, treating k as unknown. Solve the resulting equation for k and record it in Calculations Table 3. Also record the value of the correlation coefficient of the least squares fit in Calculations Table 3.
5. Calculate the percentage difference between the value of k determined indirectly by this procedure and the value of k determined earlier by measuring the elongation per unit force.

GRAPHS

1. Graph the data from Calculations Table 1 for force mg versus displacement x with mg as the ordinate and x as the abscissa. Also show on the graph the straight line obtained from the linear least squares fit to the data.
2. Graph the data from Calculations Table 2 for the period T versus the mass m with T as the ordinate and m as the abscissa.
3. Graph the data from Calculations Table 3 for the period T versus $\sqrt{m + (m_s/3)}$ with T as the ordinate and $\sqrt{m + (m_s/3)}$ as the abscissa. Also show on the graph the straight line obtained from the linear least squares fit to the data.

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Laboratory 20

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LABORATORY REPORT

Data Table 1

m (kg)	x (m)

Calculations Table 1

mg (N)	k (N/m)
Corr. Coeff. =	

Data Table 2

A (m)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)

Calculations Table 2

$\overline{\Delta t}$ (s)	α_t (s)	T (s)

SAMPLE CALCULATIONS

Data Table 3

m (kg)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)
$m_s =$ kg			

Calculations Table 3

m (kg)	$\overline{\Delta t}$ (s)	a_t (s)	T (s)	$\sqrt{m + (m_s/3)}$ $(\sqrt{kg})$
Slope =			Corr. Coeff. =	
$k =$ N/m			% Diff =	

SAMPLE CALCULATIONS

QUESTIONS

1. Do the data for the displacement of the spring x versus the applied force mg indicate that the spring constant is indeed constant for this range of forces? State clearly the evidence for your answer, including the significance of the correlation coefficient according to the table in Appendix I for the linear least squares fit to the data in Data Table 1.

2. State clearly how the period T is expected to depend on the amplitude A. Do your data confirm this expectation?

3. There are two ways to judge the agreement of the data for the dependence of the period T on the mass m. One is the correlation coefficient for the fit of T versus the quantity $\sqrt{m + (m_s/3)}$, and the other is the agreement of the value of k determined by the fit with the value of k determined directly. State as quantitatively as possible how well your data show the expected dependence of the period T on the mass m.

4. Instead of considering the spring mass m_s as known, treat it as unknown. Take the value of k determined from the elongation per force in Data Table 1 as the known value of k. Using the measured value of the period T when $m = 0.5000$ kg in equation 4 calculate a value for the spring mass m_s. Do this same calculation for the period measured when $m = 0.0500$ kg. Compare each of these values of m_s with the known value of the spring mass.

5. Generally neither of the results of question 4 will be very good, but the smaller mass will usually give the best agreement with the known spring mass. Suggest why this is not a very good measure of the spring mass and why the smaller mass is expected to be the best value.

6. The measurements of the period T were done by measuring the time for 10 periods. Why is the time for more than one period measured? If there is an advantage to measuring for 10 periods, why not measure for 1000 periods? Other than the fact that it would take too long, is there a valid reason why measuring for 1000 periods is not a good idea?