Name ________________________ Section ________ Date ________________

Laboratory 16

Centripetal Acceleration of an Object in Circular Motion

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. If a particle moves in a circle of radius R at constant speed v, its acceleration is (a) directed toward the center of the circle, (b) equal to v^2/R, (c) caused by the fact that the direction of the velocity vector changes continuously, or (d) all of the above are true.

2. If a particle moves in a circle of radius $R = 1.35$ m at a constant speed of $v = 6.70$ m/s, what is the magnitude and direction of its centripetal acceleration?

3. If the mass of the particle in question 2 is 0.350 kg, what is the magnitude and direction of the centripetal force on it?

4. A 0.500-kg particle moves in a circle of radius $R = 0.150$ m at constant speed. The time for 20 complete revolutions is 31.7 s. What is the period T of the motion? What is the speed of the particle?

5. What is the centripetal acceleration of the particle in question 4? What is the centripetal force on the particle?

6. For the apparatus used in this laboratory, the centripetal force applied to any mass is the same for a fixed radius R of rotation. Why is that statement true for this apparatus? (*Hint*: What provides the centripetal force on the rotating mass for this apparatus?)

7. A mass of 0.450 kg rotates at constant speed with a period of 1.45 s at a radius R of 0.140 m in the apparatus used for this laboratory. What is the rotation period for a mass of 0.550 kg at the same radius in this apparatus?

Centripetal Acceleration of an Object in Circular Motion

OBJECTIVES

A mass M that moves in a circle of radius R at constant speed v has a centripetal acceleration of magnitude v^2/R directed toward the center of the circle. The centripetal force F acting on the mass is therefore given by $F = Mv^2/R$. In this laboratory, independent measurements of the quantities F, M, v, and R will be made to verify that relationship. The following measurements will be made to accomplish this objective.

1. The period T of an object of mass M that rotates at constant speed v in a circle of radius R will be measured.
2. The speed v of the object will be determined from its measured period T and the measured value of R, the radius of the motion.
3. The centripetal force F will be measured directly. It is provided by a spring. The force will be determined by measuring the force required to stretch the spring when the apparatus is not rotating by the same amount it was stretched while it was rotating.

EQUIPMENT LIST

1. Hand-operated centripetal force apparatus (The device described is available from Sargent-Welch Scientific Company. A similar version is available from Central Scientific Company.)
2. Laboratory balance, calibrated slotted masses, and mass holder
3. Laboratory timer, and metal ruler

THEORY

When an object moves in a circle at constant speed the velocity vector of the object's motion is always tangent to the circle. This implies that the direction of the velocity is continuously changing, and thus the object is accelerated because acceleration is by definition a change in velocity per unit time. Figure 16.1 shows the velocity vector at various points around the circle for an object moving in a circle at constant speed. The lengths of the vectors are the same because the speed is constant, and the direction of the vectors indicate the direction of the velocity at that point. Also shown in Figure 16.1 are the velocity vectors $\mathbf{v}_i$ and $\mathbf{v}_f$ at two times t_i and t_f, with a very short time interval between them. In the third part of the figure is shown the vector difference $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$, indicating that the change in velocity $\Delta\mathbf{v}$ always points toward the center of the circle. The acceleration $\mathbf{a}$ is defined by

$$\mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t} \qquad (1)$$

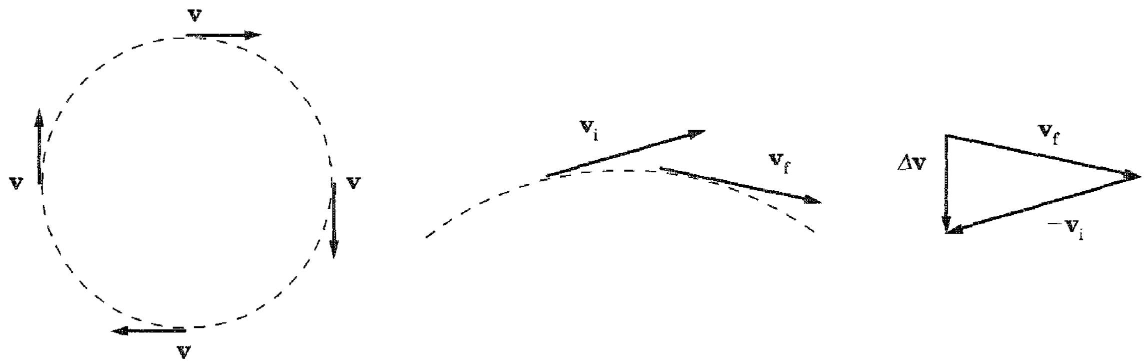

Figure 16.1 Velocity vectors for circular motion at constant speed. Vectors at two times t_i and t_f close together and the change in velocity $\Delta \mathbf{v}$ pointing toward the center of the circle.

Thus, the acceleration is in the direction of $\Delta \mathbf{v}$ and is also always pointed toward the center of the circle. The magnitude of the acceleration **a** is given by

$$a = \frac{v^2}{R} \tag{2}$$

According to Newton's second law, the magnitude of the centripetal force F and the magnitude of the centripetal acceleration a are related by $F = Ma$, where M is the mass of the object moving in a circle at constant speed v. Therefore, using equation 2 for the acceleration gives

$$F = M\frac{v^2}{R} \tag{3}$$

If the object moves at constant speed v in a circle of radius R, the time for one complete revolution around the circle is the period T. The period T is related to the speed v by the expression

$$v = \frac{2\pi R}{T} \tag{4}$$

The centripetal force apparatus has a mass bob with a pointed tip at the bottom that is suspended from a horizontal rotating bar. The bob also has a spring hooked between the side of the bob and the central rotating shaft in such a way that the spring provides a horizontal centripetal force when the bob rotates in a horizontal plane. The bob is rotated at a fixed radius R from the central rotating shaft by ensuring that the tip of the bob passes on each revolution precisely over a pointer located a distance R from the central rotating shaft. For a given mass M of the bob and a particular spring, the bob will rotate at a given radius R only for one particular rotation period T. Figure 16.2(a) shows the apparatus when the system is rotating at the period necessary to produce a rotation at radius R. A measurement of T will be made for a given R and M. Equation 4 allows a determination of v, and using that value in equation 3 allows a determination of the centripetal force F. This will be referred to as F_{theo} for the theoretical value of the force.

The force, which the spring exerts on the bob while it is rotating at the distance R from the central shaft, depends on the amount the spring is stretched under those conditions. The value of this force can be measured by determining the force needed to stretch the spring by the same amount when the apparatus is not rotating.

Figure 16.2(b) shows how a string can be attached to the other side of the bob and slotted masses can be applied over the pulley mounted near the end of the base. The weight of the total mass needed to stretch the spring until the tip of the bob is aligned with the pointer is the experimental value of the centripetal force F. This will be referred to as F_{exp}.

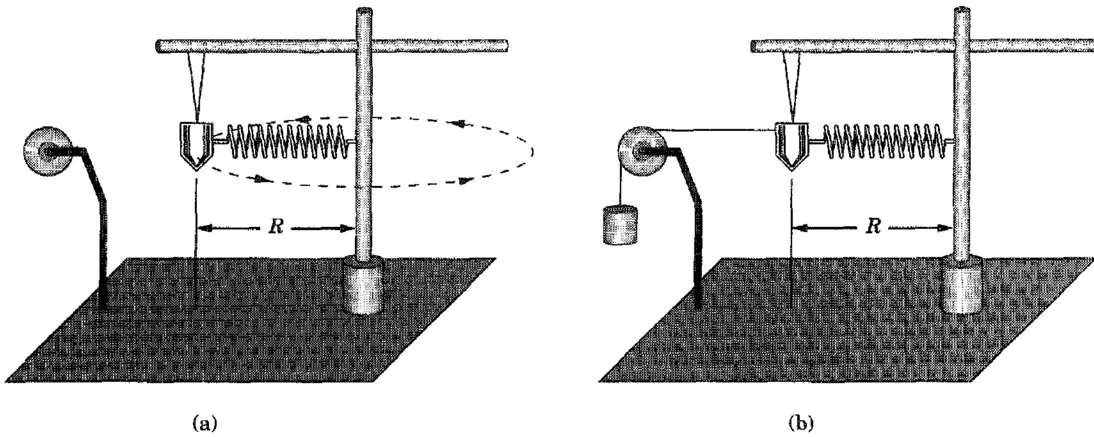

Figure 16.2 (a) Centripetal force apparatus rotating. (b) Determination of the centripetal force by measuring the force needed to stretch the spring under static conditions.

EXPERIMENTAL PROCEDURE

1. Detach the bob from its support strings and determine its mass with a laboratory balance. Be sure to remove the spring from the bob before determining its mass. Record the value of the mass of the bob as m_b in the Data Table.
2. Hang the bob from its cross-arm support by the strings that support it (Figure 16.3). Do not attach the spring to the bob but let it hang vertically. Adjust the position of the pointer to its closest position to the rotating shaft for the minimum value of R. Loosen the screw holding the cross arm in the rotating shaft and adjust its position until the tip of the bob is precisely above the tip of the pointer that determines R. The tip of the bob should be about 1 mm above the pointer. Measure with the metal ruler the distance from the center of the pointer to the center of the rotating shaft. Record this value as R in the Data Table.
3. Attach the spring to the bob and to the rotating shaft. Rotate the system as shown in Figure 16.2 (a) by twirling the rotating shaft between your thumb and first finger. The bob will pass over the position of the pointer at radius R only for one particular rotation period T. When this rotation rate has been achieved, measure the total time for 25 complete revolutions of the bob at this radius R and record it in the Data Table as Time 1. Note that you must continue to rotate the apparatus by hand while attempting to keep the rotation speed as constant as possible and at the same time ensuring that the radius of rotation is fixed and the bob passes over the pointer on each rotation. Repeat this process two more times, recording the two other measurements of the time for 25 complete revolutions as Time 2 and Time 3. Record in the Data Table the value of the rotating mass for this part of the procedure as the value of m_b.

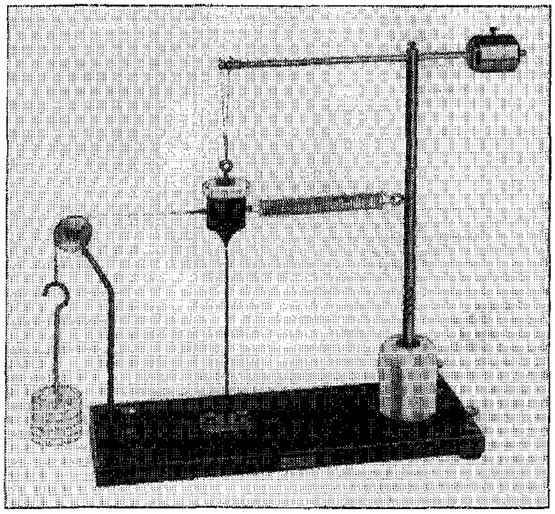

Figure 16-3 Centripetal Force Apparatus (Photo courtesy of Sargent Welch Scientific Co.).

4. With the system not rotating, measure directly the centripetal force by attaching a string to the side of the bob opposite the spring. Apply slotted weights over the pulley mounted at the end of the base as shown in Figure 16.2 (b) until the tip of the bob is just above the tip of the pointer. Let m_a stand for the total mass needed to stretch the spring by the proper amount. The experimental value for the centripetal force is then $m_a g$. Record in the Data Table the value of m_a needed to stretch the spring to the position R at which the pointer is located.
5. Repeat steps 3 and 4 above using the same value of R but using two additional values of the rotating mass. First add a 0.0500-kg slotted mass to the bob. Place the slotted mass on the top of the bob with the open end of the slot outward and secure it in place with the knurled nut on the bob. Record the value of the rotating mass in the Data Table as m_b + 0.0500 kg. Perform all the measurements of steps 3 and 4 on this mass and record all the results in the Data Table. Finally, remove the 0.0500-kg mass from the bob and replace it with a 0.1000-kg slotted mass. Record this value of the rotating mass and repeat the measurements of steps 3 and 4. Record all the results in the Data Table.
6. Remove the slotted mass from the bob. Repeat steps 3 and 4 using the bob as the rotating mass but use three different values of R. Remove the spring from the bob and let it hang vertically. Adjust the position of the pointer until it is about 1 cm further from the rotating shaft. Loosen the screw holding the cross arm and adjust it until the bob is just above the pointer at its new position. Measure the new value for R and record it in the Data Table along with the rotating mass, which is again m_b. Reattach the spring and perform the measurements of step 3 and 4 and record those results in the Data Table. Repeat the entire process for two other values of R, increasing the value of R about 1 cm each time if possible. If there is not enough range in the pointer to increase by 1 cm each time, make somewhat smaller increases in R each time. In any case, choose a total of three other values for R within the range of the variation of the pointer. Record all results in the Data Table.

CALCULATIONS

1. Calculate the mean of the three trials of the time for 25 complete revolutions and record it in the Calculations Table as $\overline{\text{Time}}$. Divide the value of $\overline{\text{Time}}$ by 25 and record the result in the Calculations Table as the period T.
2. From the measured values of R and T calculate the speed v for each case using equation 4. Record the results in the Calculations Table.
3. From the values of M, v, and R, calculate the theoretical value for the centripetal force using equation 3. Record the results in the Calculations Table as F_{theo}.
4. From the values of m_a for each case, calculate the experimental value for the centripetal force as $m_a g$. Use a value of 9.80 m/s^2 for g. Record the result in the Calculations Table as F_{exp}.
5. Calculate the percentage difference between the values of F_{theo} and F_{exp}. Record the results in the Calculations Table.

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Laboratory 16

Centripetal Acceleration of an Object in Circular Motion

LABORATORY REPORT

Data Table

m_b = ______________ kg

Rot. Mass (kg)	R (m)	Time 1 (s)	Time 2 (s)	Time 3 (s)	m_a (kg)

Calculations Table

Mass (kg)	R (m)	$\overline{\text{Time}}$ (s)	T (s)	v (m/s)	F_{theo} (N)	F_{exp} (N)	% diff

SAMPLE CALCULATIONS

QUESTIONS

1. Do your results confirm the theoretical relationship for the centripetal acceleration given by $F = Mv^2/R$? Consider the agreement between F_{theo} and F_{exp} in your answer to this question.

2. Because the centripetal force is provided by a spring for this apparatus, the centripetal force at a given distance R is fixed by the spring constant of the spring. This implies that Mv^2 is a constant for a given radius R. Calculate the quantity Mv^2 for your data taken at the same radius R. Do your results confirm this expectation?

3. Equation 3 can be written in the form $v^2 = (1/M)FR$. For a constant value of M, this would imply that the quantity v^2 should be proportional to the quantity FR, with the reciprocal of the mass as the constant of proportionality. For the four data points taken with the same mass, perform a linear least squares fit to the data with v^2 as the ordinate and FR as the abscissa. Use the values of F_{exp} in your calculations of FR. Calculate the slope of the fit and compare it to the reciprocal of the mass. Also calculate the correlation coefficient r.

4. Suppose a spring with a larger spring constant was used in this same apparatus. If a given mass were rotated at the same radius at which it was rotated with the other spring, would the new period of rotation using the new spring be greater, or would it be less than the period of rotation using the original spring?