Conservation of Spring and Gravitational Potential Energy

OBJECTIVES

- ❑ Determine the value of the spring constant k for a spring.
- ❑ Investigate the change in gravitational energy $\Delta U_g = mg(x_f - x_i)$ and the change in spring potential energy $\Delta U_k = (½)k(x_f^2 - x_i^2)$ for a mass suspended from the spring.
- ❑ Evaluate the extent to which the changes in energy are equal as the mass oscillates.

EQUIPMENT LIST

- Spring in the form of a truncated cone made of spring brass with a spring constant of about 10 N/m. (Available from Central Scientific Co. If another spring is used, appropriate adjustments should be made in the masses and distances used.)
- Set of calibrated hooked masses
- Table clamp, right angle clamps, support rods, meter stick

THEORY

When a spring is stretched or compressed a distance x from its equilibrium length, the spring exerts a restoring force F. The equation relating the force F and the displacement x is

$$F = -kx \qquad \text{(Eq. 1)}$$

and k is a constant called the **spring constant** with units N/m. The negative sign in Equation 1 indicates that the restoring force direction is opposite the displacement.

When a spring is compressed or stretched by x it has stored energy called **spring potential energy** given by $U_k = ½kx^2$. When the spring is stretched from a displacement of x_1 to a displacement of x_2 the change in spring energy is equal to the work done on the spring (Figure 12-1).

$$\text{Work} = \Delta U_k = ½\, k(x_2^2 - x_1^2) \qquad \text{(Eq. 2)}$$

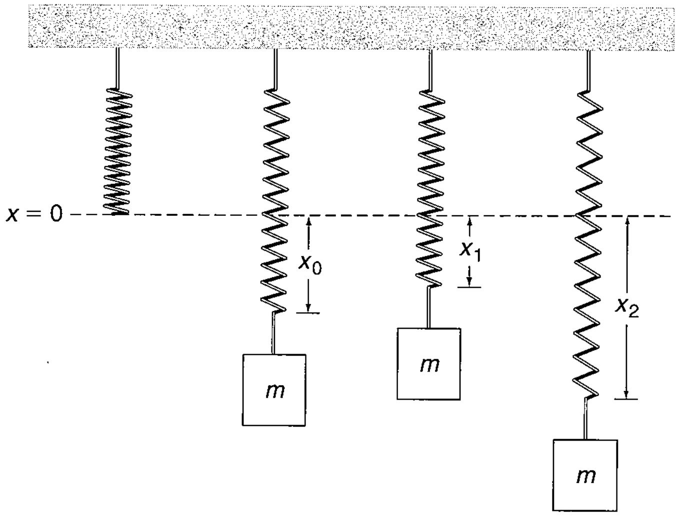

Figure 12-1 Positions of a mass m on a spring in the earth's gravitational field.

A spring with spring constant k is supported at the top by a rigid support and allowed to hang vertically. The vertical position at which the lower end of the spring hangs is the zero of the coordinate x. A hooked mass m is placed by hand on the end of the spring, and the mass is slowly lowered by hand. The mass will extend the spring by an amount x_o when the hand is removed as shown in Figure 12-1.

The mass is raised and supported by hand with the lower end of the spring at position x_1 above position x_o as shown in Figure 12-1. The mass is now released and allowed to fall under the influence of the spring and the earth's gravitational force. The mass will fall to its lowest point with displacement x_2 and then rebound and oscillate.

The **gravitational potential energy** relative to any horizontal plane is mgx where x is the distance above the plane. The **total mechanical energy** of the system is the sum of kinetic energy, spring potential energy, and gravitational potential energy. Consider the total mechanical energy at x_1 and x_2. At each of these points the kinetic energy is zero, and the total mechanical energy is the sum of the spring potential energy and the gravitational potential energy. The center of mass of m is distance d below the lower end of the spring, and the gravitational potential energy zero is the same as the equilibrium point of the spring. The equation for the sum of spring energy and gravitational energy is

$$\tfrac{1}{2} kx_1^2 - mg(x_1 + d) = \tfrac{1}{2} kx_2^2 - mg(x_2 + d) \qquad \text{(Eq. 3)}$$

$$\text{or} \quad \tfrac{1}{2} kx_1^2 - mgx_1 = \tfrac{1}{2} kx_2^2 - mgx_2 \qquad \text{(Eq. 4)}$$

Both gravitational potential energy terms are negative because the mass is below the reference point for both positions. Equation 4 can be rewritten as

$$mg(x_2 - x_1) = \tfrac{1}{2} k(x_2^2 - x_1^2) \qquad \text{(Eq. 5)}$$

Equation 5 states that the change in gravitational energy between points 1 and 2 is equal to the change in spring potential energy between those points because the kinetic energy is zero at both points 1 and 2. This laboratory will consist of a series of measurements that will test the validity of Equation 5.

EXPERIMENTAL PROCEDURE

Spring Constant

1. Use appropriate clamps and rods to provide a horizontal rod sticking out beyond the table as shown in Figure 12-2. Hang the spring on the horizontal rod, and attach it to the rod with a piece of tape.

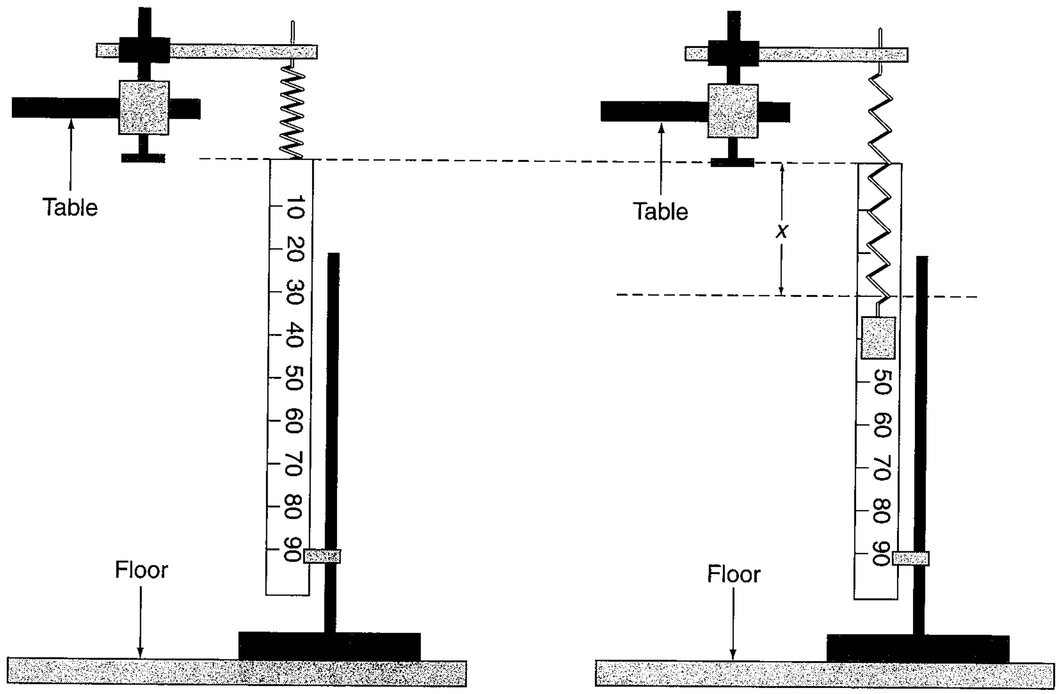

Figure 12-2 Spring supported by table clamp and meter stick aligned with the lower end of the spring. Mass placed on the end of the spring caused displacement x of the spring.

Arrange a clamp for the meter stick so that it can be supported from the floor as shown. Adjust the height of the meter stick until the zero mark of the meter stick is aligned with the bottom of the hanging spring as shown.

2. Place a hooked mass m of 0.1000 kg on the end of the spring. Slowly lower the mass m until it hangs at rest in equilibrium when released. Carefully read the position of the lower end of the spring on the meter stick scale. Record the value of the mass m and the value of the displacement x in Data Table 1.
3. Repeat Step 2, placing in succession 0.2000, 0.3000, 0.4000, and 0.5000 kg on the spring and measuring the displacement x of the spring. Record all values of m and x in Data Table 1. Record x to the nearest 0.1 mm.

Energy Conservation

1. Check that the lower end of the spring is still precisely at the zero mark. Adjust the meter stick if necessary. Hang a 0.5000 kg mass on the end of the spring and support it with your hand with the lower end of the spring precisely at the 0.2500 m mark. Record 0.2500 as x_1, and record the value of the mass in Data Table 2. Release the mass and mark the lowest point of the lower end of the spring. Release the mass several times until you have accurately located the lowest point of the motion. It may be easier to note the lowest position of the mass itself, and then hold the mass at that position to determine the position of the lower end of the spring. Record the distance as x_2 in Data Table 2.
2. Repeat Step 1 for x_1 values of 0.3000, 0.3500, and 0.4000 m. Measure the value of x_2 for each of these values of x_1 and record the values of x_1 and x_2 in Data Table 2.
3. Check that the lower end of the spring is still precisely at the zero mark. Adjust the meter stick if necessary. Use a mass of 0.5000 kg and pull the mass down by hand until the lower end of the spring is precisely at the 0.7500 m mark. Record 0.7500 m as x_2 in Data Table 3. Release the mass and determine how high it rises. The position of the lower end of the spring when the mass is at its highest point is x_1. Again release the mass several times to accurately determine the value of x_1. Record the value of x_1 and the value of the mass in Data Table 3.

4. Repeat Step 3 for x_2 values of 0.7000, 0.6500, and 0.6000 m. Measure the value of x_1 for each of these values of x_2 and record the values of x_1 and x_2 in Data Table 3.

CALCULATIONS

Spring Constant

1. Calculate the force mg for each mass and record the values in Calculations Table 1. Use the value of $9.800\ \text{m/s}^2$ for g.
2. Perform a linear least squares fit with mg as the vertical axis and x as the horizontal axis. Record the slope in Calculations Table 1 as the spring constant k and record r the correlation coefficient.

Energy Conservation

1. For each of the four measurements of the falling mass in Data Table 2, calculate the change in the gravitational potential energy ΔU_g where $\Delta U_g = mg(x_2 - x_1)$. Calculate the change in spring potential energy ΔU_k where $\Delta U_k = (1/2)k(x_2^2 - x_1^2)$. Record the results in Calculations Table 2.
2. Calculate the percentage differences between ΔU_g and ΔU_k for each case of Step 1 and record them in Calculations Table 2.
3. For each of the four measurements of the rising mass in Data Table 3, calculate the change in gravitational potential energy ΔU_g and the change in spring potential energy ΔU_k. Record the results in Calculations Table 3.
4. Calculate the percentage differences between ΔU_g and ΔU_k for each case of Step 3 and record them in Calculations Table 3.

GRAPHS

1. Graph the data from Calculations Table 1 for force mg versus displacement x with mg as the vertical axis and x as the horizontal axis. Also show on the graph the straight line obtained from the fit to the data.

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PRE-LABORATORY ASSIGNMENT

1. A spring has a spring constant of $k = 7.50$ N/m. If the spring is displaced 0.550 m from its equilibrium position, what is the force that the spring exerts? Assume for this and for all other questions in the pre-laboratory that $g = 9.80$ m/s^2. Show your work.

2. A spring of spring constant $k = 8.25$ N/m is displaced from equilibrium by a distance of 0.150 m. What is the stored energy in the form of spring potential energy? Show your work.

3. A spring of spring constant $k = 12.5$ N/m is hung vertically. A 0.500 kg mass is then suspended from the spring. What is the displacement of the end of the spring due to the weight of the 0.500 kg mass? Show your work.

4. A mass of 0.400 kg is raised by a vertical distance of 0.450 m in the earth's gravitational field. What is the change in its gravitational potential energy? Show your work.

5. A spring of spring constant $k = 8.75\,\text{N/m}$ is hung vertically from a rigid support. A mass of 0.500 kg is placed on the end of the spring and supported by hand at a point so that the displacement of the spring is 0.250 m. The mass is suddenly released and allowed to fall. At the lowest position of the mass what is the displacement of the spring from its equilibrium position? (*Hint*—Apply Equation 5 with $x_1 = 0.250\,\text{m}$ and x_2 the unknown. This will lead to a quadratic equation with one of the solutions the unknown x_2, and the other solution the original 0.250 m displacement.) Show your work.

6. The laboratory is based on the assumption that at the two points of the motion being considered, the mass is at rest. What kind of energy does not need to be included under these experimental conditions?

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LABORATORY REPORT

Data Table 1

m (kg)	x (m)

Calculations Table 1

mg (N)	k (N/m)	r

Data Table 2

x_1 (m)	x_2 (m)	m (kg)

Calculations Table 2

$mg\,(x_2 - x_1)$ (J)	$1/2k\,(x_2^2 - x_1^2)$ (J)	% *Diff*

Data Table 3

x_1 (m)	x_2 (m)	m (kg)

Calculations Table 3

$mg\ (x_2 - x_1)$ (J)	$1/2k\ (x_2^2 - x_1^2)$ (J)	% *Diff*

SAMPLE CALCULATIONS

1. $mg =$
2. $mg(x_2 - x_1) =$
3. $1/2k(x_2^2 - x_1^2) =$
4. $\%\ \text{Diff} = 2(E_1 - E_2)/(E_1 + E_2) \times 100\% =$

QUESTIONS

1. For five data points, statistical theory states that there is only 0.1% probability that a value of $r \geq 0.992$ would be obtained for uncorrelated data. Based on your value of r, make the best statement you can about the extent to which your data indicate that the force and displacement are linear.

2. Describe the extent to which your data indicate that mechanical energy is conserved in this laboratory. Consider the percentage differences in the energy changes in Data Table 2 and Data Table 3 in your answer.

3. Examine your data in Data Table 2 and in Data Table 3. For the data with the smallest percentage difference, compare the total energy at each point. Calculate the sum $U_k + U_g$ at x_1 as $\frac{1}{2}kx_1^2 - mgx_1$. Calculate that sum at x_2 as $\frac{1}{2}kx_2^2 - mgx_2$. Do you expect them to agree reasonably well? Explain why they should or should not be the same.

4. Consider the same data as used in Question 3. Calculate the value of x halfway between x_1 and x_2. Calculate $U_k + U_g = \frac{1}{2}kx^2 - mgx$ for that point. Do you expect them to agree with the energy calculated in Question 3? If they agree reasonably well, explain why they do. If they do not agree, explain why they do not agree.