# Vibration CBDC: Oscillations of a Mass on a Spring

## Background

Springs present an everyday example of forces and accelerations which are not constant but, instead, vary over time. A mass bouncing up and down on a vertically hanging spring is an important example of a oscillating or vibrating system.

In this experiment you will study the spring force. You will also study the oscillatory motion that results when a mass hanging at the end of a spring is displaced from its "resting" position and then released. After being released, a mass on the end of a spring will **oscillate**, moving up and down in a repetitive motion. The motion of the mass is determined by the interaction of three forces: gravity (the weight of the mass); the "restoring force" exerted by the coils of the spring; and friction, which gradually slows the mass to a halt after many up/down cycles. The size of the restoring force exerted by a spring is determined by two factors: the stiffness of the spring and how far the spring has been stretched or compressed from its natural or equilibrium length. The stiffness of an individual spring is expressed using a number called the "**spring constant**", symbolized by the letter ***k***. The higher the spring constant's numerical value, the stiffer the spring. That is,

$$F_{spring} = kX$$

where ***X*** is the amount the spring is stretched or compressed relative to its resting or **equilibrium** position. In the first part of this experiment you will determine the value of *k* for your spring by measuring the amount the spring is stretched when various known forces are applied to it.

The time it takes for a complete oscillation (one complete up/down motion) to occur is called the **period** of the motion and is represented with the symbol ***T***. The amount of mass attached to the spring impacts the period, as does the stiffness of the spring (the spring constant). Specifically,

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

The spring itself has some mass as well, but it is fairly small, so we will ignore it for the sake of simplicity. The second part of this experiment examines the impact mass has on period. You will investigate the effect of mass on the behavior of the spring-mass system by observing oscillations with two different masses. The period of an oscillation is related to another important quantity- frequency. **Frequency** is the number of complete "round-trip" cycles per second. It is expressed in units of Hertz, abbreviated Hz. One Hz is the same as one "cycle per second". The frequency, ***f*** is the reciprocal of the period.

$$f = \frac{1}{T}$$

## What is the Spring Constant for Your Spring?

This is the first question that we will attempt to answer. We will use the relation $F_{spring} = kX$, measuring the amount the spring is stretched from its equilibrium (or starting) position (***X***) when a known force (the weight of an object) is applied to the spring. Because the mass will be at rest when we measure its position, the acceleration of the mass is zero. This means that the downward pull of gravity (the object's weight) must be equal to the upward force from the spring and so $F_{spring} = W_{\text{hanging mass}}$.

**Set-up**

Open the LoggerPro by clicking on the icon on the desktop. The file that opens should have both a position and a velocity graph. Place the empty hanger on the spring. Place the motion detector on the floor under the hanger.

**Procedure**

**1.** Click the "collect" button at the top of the screen and collect positon data for the stationary hanger. (Does the graph make sense? Be sure the motion detector is picking up the bottom of the hanger.) Find the average position of the bottom of the hanger by using the statistics function (stat icon at the top of the screen).

**2.** Record the average position of the hanger here:

Position of the bottom of the empty mass hanger ______________________ (record in units of meters)

This will be your equilibrium or starting position.

**3**. Add a 100 gram mass to the hanger. Repeat step 1. Determine the precise mass of the 100 gram mass using the digital scale. Record your measured mass and average position in the data table below. (You're not including the mass of the hanger in this step because this was taken into account when determining our equilibrium position above.)

**4**. Repeat step 3 four more times (up to a total of 500 grams of mass added to the hanger).

| Total Mass on the Hanger (in KILOGRAMS) | Position (in METERS) |
|---|---|
| | |
| | |
| | |
| | |
| | |

**5.** Open EXCEL. Enter your measured masses and positions into a table.

**6.** Use EXCEL to calculate the weight associated with each mass. Recall that weight is mass x 9.803 ($m/s^2$). To have EXCEL do this calculation, first determine the cell location of your first mass. The location will be something like A2 where the letter signifies the column and the number signifies the row. Next, click on an empty cell that is in the same row as your first mass but in a blank column (perhaps just to the right of the position value). Next, enter the following:

**=9.803*A1** (replacing A1 with the actual location for your first mass measurement).

Hit enter. Be sure that the result reported makes sense.

**7.** Copy the formula down for each of your five measurements so that you have weight calculations for each one. (Don't reenter the formula. Figure out how to copy it down).

**8.** Use Excel to calculate the change in position or "stretch" (***X***) caused by the added weight for each of the five mass amounts. The change in position would be the position of the empty hanger minus the measured position for that mass. To do this calculation click on the cell next to your first weight calculation and enter an appropriate formula.

**9**. Copy the formula down for each of your five measurements so that you have changes in position for each. (Don't reenter the formula. Copy it down.)

**10.** Insert an x-y scatter graph of Weight vs. Change in Position (weight goes on the vertical axis). Be sure to label the axes on the graphs. Insert a trend line and show the equation of the line on the graph. Print one copy of the graph and one copy of the excel spreadsheet for each group member.

**11.** For a spring, the size of the force applied to the spring is proportional to the amount it stretches:

$$F_{spring} = kX \quad .$$

X is the amount the spring is stretched (the change in position). For this experiment, the force on the spring is the weight of the mass. How can you get the value of k (the spring constant) from your graph?

**12.** Record the value of k for your spring here.

k= ___________________________Newtons/meter

## How Does Mass Impact Period of Oscillation?

That is the next question we will attempt to answer. We will do so by using LoggerPro to measure the position of the mass over a time period as the mass oscillates.

**Set-up**

Put enough mass on the hanger so that the total mass hanging from the spring is 300 g. (If using slotted masses, some may not fit on the hanger unless they're slid down onto it from above the hook. Of course, the hanger itself has mass too!) Alternate the position of the slots so masses are less likely to fall off.

Once you have 300 grams total mass, pull the mass down just a bit, maybe a centimeter or so, and let it go. The mass should move up and down in a repetitive way that is called **oscillating**. Practice pulling the mass down and getting it oscillating until you can do so without causing it to bounce, have masses fall off or start swinging side to side.

Once you get good at starting smooth oscillations, start an oscillation and then click the collect button on the software and begin collecting data. Move the motion detector as needed to make sure that it is picking up the mass as it moves up and down.

**Procedure**

13. Pull the spring down just a little bit, maybe just a centimeter or so, and let it go. Click collect and begin collecting data. Let the mass oscillate up and down for at least 5 recorded cycles (complete up and down "round trips").
14. We want to get a precise measurement of the time it takes for 5 consecutive up/down cycles of the mass. The highest points on your graph can be taken as the start and end points for a cycle. Using high points, identify on your graph the start and end points for a run of 5 consecutive cycles. Use the examine feature of the software (x=) to get the precise start time for your first cycle and end time for your 5$^{th}$ cycle. Record your measured values in the data table below and calculate the time it takes for the entire 5 consecutive cycle run (end time-start time). Record this time in the table.
15. Repeat steps 13 and 14 four more times. Try to keep the distance that you pull the mass down about the same for all the trials. **However, changing the amount you pull down the mass does not change the amount of time it takes for 5 cycles.** Record your values for the four additional trials. Calculate the time for 5 cycles in each case (end time-start time). Calculate the average time it takes for 5 cycles.

| Trial | Start Time (seconds) | End Time (seconds) | Time for 5 cycles (end time-start time) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

Average time for 5 cycles___________________________________seconds

**16.** Calculate the period for this oscillation. The period is the time it will take for ONE up/down cycle. (Consider the number of significant figures that you want to report!) Show your work and report and answer here.

**17.** Calculate the frequency of oscillation. Frequency is the number of cycles that occur in 1 second. Frequency (*f*) is related to period (*T*) according to the equation $f = \frac{1}{T}$ . (Consider the number of significant figures you want to report!) Show your work below.

**18.** The theoretical formula for period (***T***) of an oscillating mass/spring system is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where ***m*** is the total mass and ***k*** is the spring constant for your spring. Use this expression along with the applicable mass (in KILOGRAMS) and spring constant to calculate the theoretically predicted period. Show your work.

**19.** Calculate the percent difference between this theoretical calculation of period and your average measured value from #16. The % difference between ***T*** experimental and ***T*** theoretical should be less than 10%, otherwise find the errors in your experiment and/or your theoretical calculation.

## Extension

**If you still have time left in lab you must do the additional following measurements.**

**20.** Hang a total of 1200 g from the end of the spring coil. Then, start it oscillating. Click collect and measure the time for 5 cycles by repeating steps 13-15. . **(If the correct number of peaks do not appear on your screen to measure the time for 5 cycles, you may measure using 4 cycles but be sure to divide by 4 and not 5 when calculating the period later!)**

| Trial | Start Time (seconds) | End Time (seconds) | Time for 5 cycles |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

Average time for 5 cycles ___________________________________seconds

Calculate the period and frequency:

**21.** You quadruple (x4) the mass when you go from 300 to 1200 grams. What happened to the period? How would you say that the period seems to be related to the mass?

**22.** Does the system vibrate more rapidly with a large mass or with a smaller mass?

**23.** Although it would damage the spring to hang much more than 1200 g from it, what would you predict theoretically the period and frequency would be if the mass were 4,800 g? Judge by looking at what happened to T and f when you quadrupled the mass from 300 to 1200g experimentally, and then assume the effect repeats itself. Give a numeric answer.